

THE MONIST

ON THE FOUNDATIONS OF GEOMETRY.¹

ALTHOUGH I have already had occasion to set forth my views on the foundations of geometry,¹ it will not, perhaps, be unprofitable to revert to the question with new and ampler developments, and seek to clear up certain points which the reader may have found obscure. It is with reference to the definition of the point and the determination of the number of dimensions that new light appears to me most needed; but I deem it opportune, nevertheless, to take up the question from the beginning.

SENSIBLE SPACE.

Our sensations cannot give us the notion of space. That notion is built up by the mind from elements which pre-exist in it, and external experience is simply the occasion for its exercising this power, or at most a means of determining the best mode of exercising it.

Sensations by themselves have no spatial character.

This is evident in the case of isolated sensations—for example, visual sensations. What could a man see who possessed but a single immovable eye? Different images would be cast upon different points of his retina, but would he be led to classify these images as we do our present retinal sensations?

¹ Translated from Professor Poincaré's MS. by T. J. McCormack.

² Both in the *Revue Générale des Sciences* and in the *Revue de Métaphysique et de Morale*.

Suppose images formed at four points A , B , C , D of this immovable retina. What ground would the possessor of this retina have for saying that, for example, the distance AB was equal to the distance CD ? We, constituted as we are, have a reason for saying so, because we know that a *slight* movement of the eye is sufficient to bring the image which was at A to C , and the image which was at B to D . But these slight movements of the eye are impossible for our hypothetical man, and if we should ask him whether the distance AB was equal to the distance CD , we should seem to him as ridiculous as would a person appear to us who should ask us whether there was more difference between an olfactory sensation and a visual sensation than between an auditive sensation and a tactual sensation.

But this is not all. Suppose that two points A and B are very near to each other, and that the distance AC is very great. Would our hypothetical man be cognisant of the difference? We perceive it, we who can move our eyes, because a very slight movement is sufficient to cause an image to pass from A to B . But for him the question whether the distance AB was very small as compared with the distance AC would not only be insoluble, but would be devoid of meaning.

The notion of the contiguity of two points, accordingly, would not exist for our hypothetical man. The rubric, or category, under which he would arrange his sensations, if he arranged them at all, would consequently not be the space of the geometer and would probably not even be continuous, since he could not distinguish small distances from large. And even if it were continuous, it could not, as I have abundantly shown elsewhere, be either homogeneous, isotropic, or tridimensional.

It is needless to repeat for the other senses what I have said for sight. Our sensations differ from one another qualitatively, and they can therefore have no common measure, no more than can the gramme and the metre. Even if we compare only the sensations furnished by the same nerve-fibre, considerable effort of the mind is required to recognise that the sensation of to-day is of the same kind as the sensation of yesterday, but greater or

smaller; in other words, to classify sensations according to their character, and then to arrange those of the same kind in a sort of scale, according to their intensity. Such a classification cannot be accomplished without the active intervention of the mind, and it is the object of this intervention to refer our sensations to a sort of rubric or category pre-existing in us.

Is this category to be regarded as a "form of our sensibility"? No, not in the sense that our sensations, individually considered, could not exist without it. It becomes necessary to us only for comparing our sensations, for reasoning upon our sensations. It is therefore rather a form of our understanding.

This, then, is the first category to which our sensations are referred. It can be represented as composed of a large number of scales absolutely independent of one another. Further, it simply enables us to compare sensations of the same kind and not to measure them, to perceive that one sensation is greater than another sensation, but not that it is twice as great or three times as great.

How much such a category differs from the space of the geometer! Shall we say that the geometer admits a category of quite the same kind, where he employs three scales such as the three axes of co-ordinates? But in our category we have not three scales only, but as many as there are nerve-fibres. Further, our scales appear to us as so many separate worlds fundamentally distinct, while the three axes of geometry all fulfil the same office and may be interchanged one for another. In fine, the co-ordinates are susceptible of being measured and not simply of being compared. Let us see, therefore, how we can rise from this rough category which we may call sensible space to geometric space.

THE FEELING OF DIRECTION.

It is frequently said that certain of our sensations are always accompanied by a peculiar feeling of direction, which gives to them a geometrical character. Such are visual and muscular sensations. Others on the contrary like the sensations of smell and taste are not accompanied by this feeling, and consequently are void of any geometrical character whatever. On this theory the

notion of direction would be pre-existent to all visual and muscular sensations and would be the underlying condition of the same.

I am not of this opinion; and let us first ask if the feeling of direction really forms a constituent part of the sensation. I cannot very well see how there can be anything else *in* the sensation than the sensation itself. And be it further observed that the same sensation may, according to circumstances, excite the feeling of different directions. Whatever be the position of the body, the contraction of the *same* muscle, the biceps of the right arm, for example, will always provoke the *same* muscular sensation; and yet, through being apprised by other concomitant sensations that the position of the body has changed, we also know perfectly well that the direction of the motion has changed.

The feeling of direction, accordingly, is not an integrant part of the sensation, since it can vary without the sensation being varied. All that we can say is that the feeling of direction is associated with certain sensations. But what does this signify? Do we mean by it that the sensation is associated with a certain indescribable something which we can represent to ourselves but which is still not a sensation? No, we mean simply that the various sensations which correspond to the same direction are associated *with one another*, and that one of them calls forth the others in obedience to the ordinary laws of association of ideas. Every association of ideas is a product of habit merely, and it would be necessary for us to discover how the habit was formed.

But we are still far from geometrical space. Our sensations have been classified in a new manner: those which correspond to the same direction are grouped together; those which are isolated and have reference to no direction are not considered. Of the innumerable scales of sensations of which our sensible space was formed some have disappeared, others have been merged into one another. Their number has been diminished.

But the new classification is still not space; it involves no idea of measurement; and, furthermore, the restricted category so reached would not be an isotropic space, that is to say, different directions would not appear to us as fulfilling the same office and

as interchangeable with one another. And so this "feeling of direction" far from explaining space would itself stand in need of explanation.

But will it help us even towards the explanation we seek? No, because the laws of that association of ideas which we call the feeling of direction are extraordinarily complex. As I explained above, the same muscular sensation may correspond to a host of different directions according to the position of the body which is made known to us by other concomitant sensations. Associations so complex can only be the result of an extremely long process. This, therefore, is not the path which will lead us most quickly to our goal. Therefore we will not regard the feeling of direction as something attained but will revert to the "sensible space" with which we started.

REPRESENTATION OF SPACE.

Sensible space has nothing in common with geometrical space. I believe that few persons will be disposed to contest this assertion. It would be possible, perhaps, to refine the category which I set up at the beginning of this article, and to construct something which would more resemble geometrical space. But whatever concession we might make, the space so constructed would be neither infinite, homogeneous, nor isotropic: it could be such only by ceasing to be accessible to our senses.

Seeing that our representations are simply the reproductions of our sensations, therefore we cannot image geometrical space. We cannot represent to ourselves objects in geometrical space, but can merely reason upon them as if they existed in that space.

A painter will struggle in vain to construct an object of three dimensions upon canvas. The image which he traces, like his canvas, will never have more than two. When we endeavor, for example, to represent the sun and the planets in space, the best we can do is to represent the visual sensations which we experience when five or six tiny spheres are set revolving in close proximity. Geometrical space, therefore, cannot serve as a category for

our representations. It is not a form of our sensibility. It can serve us only in our reasonings. It is a form of our understanding.

DISPLACEMENT AND ALTERATION.

We at once perceive that our sensations vary, that our impressions are subject to change. The laws of these variations were the cause of our creating geometry and the notion of geometrical space. If our sensations were not variable, there would be no geometry.

But that is not all. Geometry could not have arisen unless we had been led to distribute into two classes the changes which can arise in our impressions. We say, in one case, that our impressions have changed because the objects causing them have undergone some alterations of character, and again that these impressions have changed because the objects have suffered displacement. What is the foundation of this distinction?

A sphere of which one hemisphere is blue and the other red, is rotating before our eyes and shows first a blue hemisphere and then a red hemisphere. Again, a blue liquid contained in a vase suffers a chemical reaction which causes it to turn red. In both cases the impression of blue has given way to the impression of red. Now why is the first of these changes classed among displacements, and the second among alterations? Evidently because in the first case it is sufficient for me merely to go around the globe to bring myself face to face again with the other hemisphere, and so to receive a second time the impression of blue.

An object is displaced before my eye, and its image which was first formed on the centre of the retina is now brought to the edge of the retina. The sensation which was carried to me by a nerve-fibre proceeding from the centre of the retina is succeeded by another which is carried to me by a fibre proceeding from the edge. These sensations are conducted to me by two different nerves. They ought to appear to me different in character, and if they did not, how could I distinguish them?

Why, then, do I come to conclude that the *same* image has been displaced? Is it because one of these sensations frequently

succeeds the other? But similar successions are frequent. These it is that produce all our associations of ideas, and we do not ordinarily conclude that they are due to displacement of an object which is invariable in character.

But what happens in this case is that we can *follow the object with the eye*, and by a displacement of our eye which is generally voluntary and accompanied by muscular sensations, we can bring the image back to the centre of the retina and so *re-establish the primitive sensation*. The following, therefore, is my conclusion.

Among the changes which our impressions undergo, we distinguish two classes:

(1) The first are independent of our will and not accompanied by muscular sensations. These are *external changes* so called.

(2) The others are voluntary and accompanied by muscular sensations. We may call these *internal changes*.

We observe next that in certain cases when an external change has modified our impressions, we can, by voluntarily provoking an internal change, re-establish our primitive impressions. The external change, accordingly, can be *corrected* by an internal change. External changes may consequently be subdivided into the two following classes:

1. Changes which are susceptible of being corrected by an internal change. These are *displacements*.

2. Changes which are not so susceptible. These are *alterations*.

An immovable being would be incapable of making this distinction. *Such a being, therefore, could never create geometry*,—even if his sensations were variable, and even if the objects surrounding him were movable.

CLASSIFICATION OF DISPLACEMENTS.

A sphere of which one hemisphere is blue and the other red, is rotating before me and presents to me first its blue side and then its red side. I regard this external change as a displacement because I can correct it by an internal change, namely, by going around the sphere. Let us repeat the experiment with another sphere, of which one hemisphere is green and the other yellow.

The impression of the yellow hemisphere will succeed that of the green, as before that of the red succeeded that of the blue. For the same reason I shall regard this new external change as a displacement.

But this is not all. I also say that these two external changes are due to the *same* displacement, that is to say, to a rotation. Yet there is no connexion between the impression of the yellow hemisphere and that of the red, any more than there is between that of the blue and that of the green, and I have no reason for saying that the same relation exists between the yellow and the green as exists between the red and the blue. No, I say that these two external changes are due to the same displacement because I have "corrected" them by the same internal change. But how am I to know that the two internal changes by which I corrected first the external change from the blue to the red, then that from the green to the yellow, are to be considered identical? Simply because they have provoked the *same* muscular sensations; and for this it is not necessary for me to know geometry in advance and to represent to myself the movements of my body in geometric space.

Thus several external changes which in themselves have no common relation may be corrected by the same internal change. I collect these into the same class and consider them as the same displacement.

An analogous classification may be made with respect to internal changes. All internal changes are not capable of correcting an external change. Only those which are may be called displacements. On the other hand the same external change may be corrected by several different internal changes. A person knowing geometry might express this idea by saying that my body can go from the position *A* to the position *B* by several different paths. Each of these paths corresponds to a series of muscular sensations; and at present I am cognisant of nothing but these muscular sensations. No two of these series have a common resemblance, and if I consider them nevertheless as representing the *same* displacement, it is because they are capable of correcting the same external change.

The foregoing classification suggests two reflexions :

1. The classification is not a crude datum of experience, because the aforementioned compensation of the two changes, the one internal and the other external, is never exactly realised. It is, therefore, an active operation of the mind, which endeavors to insert the crude results of experience into a pre-existing form, into a category. This operation consists in identifying two changes because they possess a common character, and in spite of their not possessing it exactly. Nevertheless, the very fact of the mind's having occasion to perform this operation is due to experience, for experience alone can teach it that the compensation has approximately been effected.

2. The classification further brings us to recognise that two displacements are identical, and it hence results that a displacement can be *repeated* twice or several times. It is this circumstance that introduces number, and that permits measurement where formerly pure quality alone held sway.

INTRODUCTION OF THE NOTION OF GROUP.

That we are able to go farther is due to the following fact, the importance of which is cardinal.

It is obvious that if we consider a change A , and cause it to be followed by another change B , we are at liberty to regard the *ensemble* of the two changes A followed by B as a single change which may be written $A + B$ and may be called the resultant change. (It goes without saying that $A + B$ is not necessarily identical with $B + A$.) The conclusion is then stated that if the two changes A and B are displacements, the change $A + B$ also is a displacement. Mathematicians express this by saying that *the ensemble, or aggregate, of displacements is a group*. If such were not the case there would be no geometry.

But how do we know that the *ensemble* of displacements is a group? Is it by reasoning *a priori*? Is it by experience? One is tempted to reason *a priori* and to say: if the external change A is corrected by the internal change A' , and the external change B by the internal change B' , the resulting external change $A + B$ will be

corrected by the resulting internal change $B' + A'$. Hence this resulting change is by definition a displacement, which is to say that the *ensemble* of displacements forms a group.

But this reasoning is open to several objections. It is obvious that the changes A and A' compensate each other; that is to say, that if these two changes are made in succession, I shall find again my original impressions,—a result which I might write as follows:

$$A + A' = 0.$$

I also see that $B + B' = 0$. These are hypotheses which I made at the outset and which served me in defining the changes A , A' , B , and B' . But is it certain that we shall still have $B + B' = 0$,—after the two changes A and A' ? Is it certain that these two changes compensate in such a manner that not only shall I recover my original impressions, but that the changes B and B' shall recover all their original properties, and in particular that of mutual compensation? If we admit this, we may conclude from it that I shall recover my primitive impressions when the four changes follow in the order

$$A, A', B, B';$$

but not that the same will still be the case when they succeed in the order

$$A, B, B', A'.$$

Nor is this all. If two external changes α and α' are regarded as identical on the basis of the convention adopted above, or in other words, are susceptible of being corrected by the same internal change A ; if, on the other hand, two other external changes β and β' can be corrected by the same internal change B , and consequently may also be regarded as identical, have we the right to conclude that the two changes $\alpha + \beta$ and $\alpha' + \beta'$ are susceptible of being corrected by the same internal change, and are consequently identical? Such a proposition is in no wise evident; and if it be true it cannot be the result of *a priori* reasoning.

Accordingly, this set of propositions, which I recapitulate by saying that displacements form a group, is not given us by *a priori* reasoning. Are they then a result of experience? One is inclined to admit that they are; and yet one has a feeling of real misgiving

in so doing. May not more precise experience prove some day that the law above enunciated is only approximate? What, then, will become of geometry?

But we may rest assured on this point. Geometry is safe from all revision; no experience, however precise, can overthrow it. If it could have done it, it would have done so long ago. We have long known that all the so-called experimental laws are approximations, and rough approximations at that.

What, then, is to be done? When experience teaches us that a certain phenomenon does not correspond *at all* to these laws, we strike it from the list of displacements. When it teaches us that a certain change obeys them *only approximately*, we consider the change, *by an artificial convention*, as the resultant of two other component changes. The first component is regarded as a displacement *rigorously* satisfying the laws of which I have just spoken, while the second component, which is small, is regarded as a qualitative alteration. Thus we say that natural solids undergo not only great changes of position but also small flexions and small thermal dilatations.

By an external change α we pass, for example, from the *ensemble* of impressions *A* to the *ensemble* *B*. We correct this change by a voluntary internal change β and are carried back to the *ensemble* *A*. A new external change α' causes us to pass again from the *ensemble* *A* to the *ensemble* *B*. We ought to expect then that this change α' could in its turn be corrected by another voluntary internal change β' which would provoke the same muscular sensations as β and which would call forth again the *ensemble* of impressions *A*. If experience does not confirm this prediction, we shall not be embarrassed. We say that the change α' , although like α it has been the cause of my passing from the *ensemble* *A* to the *ensemble* *B*, is nevertheless not identical with the change α . If our prediction is confirmed only approximately we say that the change α' is a displacement identical with the displacement α but accompanied by a slight qualitative alteration.

In fine, these laws are not imposed by nature upon us but are imposed by us upon nature. But if we impose them upon nature,

it is because she suffers us to do so. If she offered too much resistance, we should seek in our arsenal for another form which would be more acceptable to her.

CONSEQUENCES OF THE EXISTENCE OF THE GROUP.

This first fact, that displacements form a group, contains in germ a host of important consequences. Space must be homogeneous; that is, all its points are capable of playing the same part. Space must be isotropic; that is, all directions which issue from the same point must play the same part.

If a displacement D transports me from one point to another, or changes my orientation, I must after such displacement D be still capable of the same movements as before the displacement D , and these movements must have preserved their fundamental properties, which permitted me to classify them among displacements. If it were not so, the displacement D followed by another displacement would not be equivalent to a third displacement; in other words, displacements would not form a group.

Thus the new point to which I have been transported plays the same part as that at which I was originally; my new orientation also plays the same part as the old; space is homogeneous and isotropic.

Being homogeneous, it will be unlimited; for a category that is limited cannot be homogeneous, seeing that the boundaries cannot play the same part as the centre. But this does not say that it is infinite; for the sphere is an unbounded surface, and yet it is finite. All these consequences, accordingly, are germinally contained in the fact which we have just discovered. But we are as yet unable to perceive them, because we do not yet know what a direction is or even what a point is.

PROPERTIES OF THE GROUP.

We have now to study the properties of the group. These properties are purely formal. They are independent of any quality whatever, and in particular of the qualitative character of the phenomena which constitute the change to which we have given the

name displacement. We remarked above that we could regard two changes as representing the same displacement, although the phenomena were quite different in qualitative nature. The properties of this displacement remain the same in the two cases; or rather the only ones which concern us, the only ones which are susceptible of being studied mathematically, are those in which quality is in no wise concerned. A brief digression is necessary here to render my thought comprehensible. What mathematicians call a group is the *ensemble* of a certain number of operations and of all the combinations which can be made of them. In the group which is occupying us our operations are displacements. It sometimes happens that two groups contain operations which are entirely different as to character, and that these operations nevertheless combine according to the same laws. We then say that the two groups are *isomorphic*.

The different permutations of six objects form a group and the properties of this group are independent of the character of the objects. If in place of the six material objects we take six letters, or even the six faces of a cube, we obtain groups which differ as to their component materials, but which are all isomorphic with one another.

The formal properties are those which are common to all isomorphic groups. If I say, for example, that such and such an operation repeated three times is equivalent to such and such an other repeated four times, I have announced a formal property entirely independent of quality. These formal properties are susceptible of being studied mathematically. They should be enunciated, therefore, in *precise* propositions. On the other hand, the experiences which serve to verify them can never be more than approximate; that is to say, the experiences in question can never be the true foundation of these propositions. We have within us, in a potential form, a certain number of models of groups, and experience merely assists us in discovering which of these models departs least from reality.

CONTINUITY.

It is observed first that the group is *continuous*. Let us see what this means, and how the fact can be established.

The same displacement can be repeated twice, three times, etc. We obtain thus different displacements which may be regarded as *multiples* of the first. The multiples of the same displacement D form a group; for the succession of two of these multiples is still a multiple of D . Further, all these multiples are interchangeable (a truth which is expressed by saying that the group which they form is a *sheaf*); that is, it is indifferent whether we repeat D first three times and then four times, or first four times and then three times. This is an analytical judgment *a priori*; an out-and-out tautology. This group of the multiples of D is only a part of the total group. It is what is called a *sub-group*.

Now we soon discover that any displacement whatever can always be divided into two, three, or any number of parts whatever; I mean that we can always find an other displacement which, repeated two, three times will reproduce the given displacement. This divisibility to infinity conducts us naturally to the notion of mathematical continuity; yet things are not so simple as they appear at first sight.

We cannot prove this divisibility to infinity, directly. When a displacement is very small, it is inappreciable for us. When two displacements differ very little, we cannot distinguish them. If a displacement D is extremely small, its consecutive multiples will be indistinguishable. It may happen then that we cannot distinguish $9D$ from $10D$, nor $10D$ from $11D$, but that we can nevertheless distinguish $9D$ from $11D$. If we wanted to translate these crude facts of experience into a formula, we should write

$$9D=10D, 10D=11D, 9D<11D.$$

Such would be the formula of physical continuity. But such a formula is repugnant to reason. It corresponds to none of the models which we carry about in us. We escape the dilemma by an artifice; and for this physical continuity—or, if you prefer, for

this sensible continuity, which is presented in a form unacceptable to our minds—we substitute mathematical continuity. Severing our sensations from that something which we call their cause, we assume that the something in question conforms to the model which we carry about in us, and that our sensations deviate from it only in consequence of their crudeness.

The same process recurs every time we apply measurement to the data of the senses; it is notably applicable to the study of displacements. From the point which we have now reached, we can render an account of our sensations in several different ways.

(1) We may suppose that each displacement forms part of a sheaf formed of all the multiples of a certain small displacement far too small to be appreciated by us. We should then have a discontinuous sheaf which would give us the illusion of physical continuity because our gross senses would be unable to distinguish any two consecutive elements of the sheaf.

(2) We may suppose that each displacement forms part of a more complex and richer sheaf. All the displacements of which this sheaf is composed would be interchangeable. Any two of them would be multiples of another smaller displacement which likewise formed part of the sheaf and which might be regarded as their greatest common divisor. Finally, any displacement of the sheaf could be divided into two, three, or any number of parts, in the sense which I have given to this word above, and the divisor would still be part of the sheaf. The different displacements of the sheaf would be, so to speak, commensurable with one another. To every one of them would correspond a commensurable number, and *vice versa*. This therefore would be already a sort of mathematical continuity, but this continuity would still be imperfect, for there would be nothing corresponding to incommensurable numbers.

(3) We may suppose, finally, that our sheaf is perfectly continuous. All its displacements are interchangeable. To every commensurable or incommensurable number corresponds a displacement and *vice versa*. The displacement corresponding to the number na is nothing else than the displacement corresponding to the number a repeated n times.

Why has the last of these three solutions been adopted? The reasons for the choice are complicated.

(1) It has been established by experience that displacements which are sufficiently large can be divided by any number whatever; and as the means of measurement increased in precision, this divisibility was demonstrated for displacements much smaller, with respect to which it first seemed doubtful. We have thus been led by induction to suppose that this divisibility is a property of all displacements, however small, and consequently to reject the first solution and to decide in favor of divisibility to infinity.

(2) The first solution, like the second, is incompatible with the other properties of the group which we know from other experience. I shall explain this further on. The third solution, accordingly, is imposed upon us by this fact alone. The contrary might have happened. It might have been that the properties of the group were incompatible with continuity. Then we should undoubtedly have adopted the first solution.

SUB-GROUPS.

The most important of the formal properties of a group is the existence of sub-groups. It must not be supposed that there can be as many sub-groups formed as we like, and that it is sufficient to cut up a group in an arbitrary manner, as one would inert clay, in order to obtain a sub-group. If two displacements be taken at random in a group, it will be necessary, in order to form a sub-group from them, to conjoin with them all their combinations; and in the majority of cases it happens that in combining these two displacements in all possible manners we arrive ultimately at the primitive group again in its original intact form. It may happen thus that a group contains no sub-group.

But groups are distinguished from one another, in a formal point of view, by the number of sub-groups which they contain and by the mutual relations of the sub-groups. A superficial examination of the group of displacements renders it patent that it contains some sub-groups. A more minute examination will disclose them all. We shall see that among these sub-groups there are some that

are: (1) continuous, i. e., have all their displacements divisible to infinity; (2) discontinuous, i. e., have no displacements that are divisible to infinity; (3) mixed, i. e., have displacements divisible to infinity and in addition others that are not so divisible.

From another point of view we distinguish among our sub-groups sheaves whose displacements are all interchangeable and those which do not possess this property.

The following is another manner of classing displacements and sub-groups.

Let us consider two displacements D and D' . Let D'' be a third displacement, defined to be the resultant of the displacement D' followed by the displacement D followed itself by the inverse displacement of D' . This displacement D'' is called the *transformation of D by D'* .

From the formal point of view all the transformations of the same displacement are equivalent, so to speak; they play the same part; the Germans say that they are *gleichberechtigt*. Thus (if I may be permitted for an instant to use in advance the ordinary language of geometry which we are supposed not yet to know) two rotations of 60° are *gleichberechtigt*, two helicoidal displacements of the same step and same fraction of spiral are *gleichberechtigt*.

The transformations of all displacements of a sub-group g by the same displacement D' form a new sub-group which is called the transformation of the sub-group g by the displacement D' . The different transformations of the same sub-group, playing the same part in a formal point of view, are *gleichberechtigt*.

It happens generally that many of the transformations of the same sub-group are identical; it will sometimes even happen that all the transformations of a sub-group are identical with one another and with the primitive sub-group. It is then said this sub-group is *invariant* (which happens, for example, in the case of the sub-group formed of all translations). The existence of an invariant sub-group is a formal property of the highest importance.

ROTATIVE SUB-GROUPS.

The number of sub-groups is infinite; but they may be divided into a rather limited number of classes of which I do not wish to give here a complete enumeration. But these sub-groups are not all perceived with the same facility. Some among them have been only recently discovered. Their existence is not an intuitive truth. Unquestionably it can be deduced from the fundamental properties of the group, from properties which are known to everybody, and which are, so to speak, the common patrimony of all minds. Unquestionably it is contained there in germ; yet those who have demonstrated their existence have justly felt that they had made a discovery and have frequently been obliged to write long memoirs to reach their results.

Other sub-groups, on the contrary, are known to us in much more immediate manner. Without much reflexion every one believes he has a direct intuition of them, and the affirmation of their existence constitutes the axioms of Euclid. Why is it that some sub-groups have directly attracted attention, whilst others have eluded all research for a much longer time? We shall explain it by a few examples.

A solid body having a fixed point is turning before our eyes. Its image is depicted on our retina and each of the fibres of the optic nerve conveys to us an impression; but owing to the motion of the solid body this impression is variable. One of these fibres, however, conveys to us a constant impression. It is that at the extremity of which the image of the fixed point has been formed. We have, thus, a change which causes certain sensations to vary, but leaves others invariable. This is a property of the displacement, but at first blush it does not appear that it is a formal property. It seems to belong to the qualitative character of the sensations experienced. We shall see, however, that we can disengage a formal property from it, and to render my thought clear I shall compare what takes place in this case with what happens in another instance which is apparently analogous.

I suppose that a certain body is moving before my eyes in any manner, but that a certain region of this body is painted in a color sufficiently uniform to leave no shades discernible. Let us say it is red. If the movements are not of too great compass and if the red region is sufficiently large in extent, certain parts of the retina will remain constantly in the image of that region, certain nerve-fibres will convey to us constantly the impression of the red, the displacement will have left certain sensations invariable.

But there is an essential difference between the two cases. Let us go back to the first one. We witnessed there an external change in which certain sensations *A* did not change, whilst other sensations *B* did change. We are able to correct this external change by an internal change, and in this correction the sensations *A* still remain invariable.

But now here is a new solid body which is turning before our eyes and is experiencing the same rotations as the first. This is a new external change which may be different altogether from the first from a qualitative point of view, because the new body which is turning may be painted in new colors, or because we are apprised of its rotation by touch and not by sight. We discover, however, that it is the *same* displacement, because it can be corrected by the same internal change. And we also discover that certain sensations *A'* in this new external change (totally different perhaps from *A*) have remained invariable, whilst other sensations *B'* varied. Thus, this property of conserving certain sensations ultimately appears to us as a formal property independent of the qualitative character of these sensations.

We pass to the second example. We have, first, an external change in which a certain sensation *C*, a sensation of red, has remained constant. Let us suppose that another solid body, differently painted, undergoes the same displacement. Here is a new external change, and we know that it represents the same displacement because we can correct it by the same internal change. We discover generally that in this new external change certain sensations have not remained constant. Thus the conservation of the

sensation *C* will appear to us as an accidental property only, connected with the qualitative nature of the sensation.

We are thus led to distinguish among displacements those which conserve certain sensations. The *ensemble* of the displacements which thus conserve a given system of sensations, evidently forms a sub-group which we may call a *rotative sub-group*.

Such is the conclusion which we draw from experience. It is needless to point out how crude is the experience and how precise on the other hand is the conclusion. Therefore experience cannot impose the conclusion upon us, but it suffices to suggest it to us. It suffices to show that of all the groups of which the models pre-exist in us, the only ones which we can accept with a view of referring to them our sensations, are those which contain such a sub-group.

By the side of the rotative sub-group, we should consider its transformations, which also may be called rotative sub-groups. (Sub-group of rotations about a fixed point.) By new experiences, always very crude, it is then shown :

- (1) That any two rotative sub-groups have common displacements.
- (2) That these common displacements, all interchangeable among one another, form a sheaf, which may be called a rotative sheaf. (Rotations about a fixed axis.)
- (3) That any rotative sheaf forms part not only of two rotative sub-groups, but of an infinity of them.

Here is the origin of the notion of the straight line, as the rotative sub-group was the origin of the notion of the point.

Let us now look at all the displacements of a rotative sheaf. If we look at any displacement whatever, it will not in general be interchangeable with all the displacements of the sheaf, but we shall discover very soon that there exist displacements which are interchangeable with all those of the rotative sheaf, and that they form a more extensive sub-group which may be called the helicoidal sub-group (combinations of rotations about an axis and of translations parallel to that axis). This will be evident when it is observed that a straight line can slide along itself.

Finally, we derive from the same crude observations such propositions as the following :

Any displacement sufficiently small and forming part of a given rotative sub-group, can always be decomposed into three others belonging respectively to three given rotative sheaves. Every displacement interchangeable with a rotative sub-group forms part of this sub-group.

Any displacement sufficiently small can always be decomposed into two others belonging respectively to two given rotative sub-groups, or to *six given rotative sheaves*.

Later on I shall revert in detail to the origin of these various propositions.

TRANSLATIVE SUB-GROUPS.

With these propositions we have sufficient material, not to construct the geometry of Euclid, but to limit the choice between that of Euclid and the geometries of Lobatchévski and Riemann. In order to go farther, we are in need of a new proposition to take the place of the postulate of parallels. The proposition substituted will be the existence of an *invariant* sub-group, of which all the displacements are interchangeable and which is formed of all translations.

It is this that determines our choice in favor of the geometry of Euclid, because the group that corresponds to the geometry of Lobatchévski does not contain such an invariant sub-group.

NUMBER OF DIMENSIONS.

In the ordinary theory of groups, we distinguish order and degree. Let us suppose the simplest case first, that of a group formed by different permutations between certain objects. The number of the objects is called the degree; the number of the permutations is called the order of the group. Two such groups may be isomorphic and their permutations may combine according to the same laws without their degree being the same. Thus let us consider the different ways in which a cube can be superposed upon itself. The vertices may be interchanged one with another, as may also be the

faces and the edges ; whence result three groups of permutations which are evidently isomorphic among themselves ; but their degree may be either eight, six, or twelve, since there are eight vertices, six faces, and twelve edges.

On the other hand, two mutually isomorphic groups have always the same order. The degree is, so to speak, a material element, and the order a formal element, the importance of which is far greater. The theory of two groups of different degree may be the same so far as its formal properties are concerned ; just as the mathematical theory of the addition of three cows and four cows is identical with that of three horses and four horses.

When we pass to continuous groups, the definitions of order and degree must be modified, though without sacrificing their spirit. Mathematicians suppose ordinarily that the object of the operations of the group is an *ensemble* of a certain number n of quantities susceptible of being varied in a continuous manner, which quantities are called *co-ordinates*. On the other hand, every operation of the group may be regarded as forming part of a sheaf analogous to the rotative sheaf and as a multiple of a very high order of an infinitesimal operation belonging to the same sheaf. Then, every infinitesimal operation of the group can be decomposed into k other operations belonging to k given sheaves. The number n of the co-ordinates (or of the dimensions) is then the *degree*, and the number k of the components of an infinitesimal operation is the *order*. Here again two isomorphic groups may have different degrees, but must be of the same order. Here again the degree is an element relatively material and secondary, and the order a formal element. According to the laws established above, our group of displacements is here of the sixth order, but its degree is yet unknown. Is the degree given us immediately?

Displacements, we have seen, correspond to changes in our sensations, and if we distinguish in the present group between form and material, the material can be nothing else than that which the displacements cause to change, viz., our sensations. Even if we suppose that what we have above called sensible space has already been elaborated, the material would then be represented

by as many continuous variables as there are nerve-fibres; the "degree" of our group would then be extremely large. Space would not have three dimensions but as many as there are nerve-fibres. Such is the consequence to which we come if we accept as the material of our group what is immediately given us. How shall we escape the difficulty? Evidently by replacing the group which is given us, together with its form and its material, by another *isomorphic* group, the material of which is simpler.

But how is this to be done? It is precisely owing to this circumstance, that the displacements which conserve certain elements are the same as those which conserve certain other elements. Then all those elements which are conserved by the same displacements we agree to replace by a single element which has a purely schematic value only. Whence results a considerable reduction of degree.

For example, I see a solid body rotating about a fixed point. The parts near the fixed point are painted red. Here is a displacement, and within this displacement I perceive that something remains invariable—namely, the sensation of red conveyed to me by a certain optical nerve-fibre. Some time afterward I see an other solid body turning about a fixed point. But the parts near the fixed point are painted green. The sensations experienced are in themselves quite different, but I perceive that it is the same displacement because it can be corrected by the same internal change. Here again something remains invariable; but this something is totally different from the material point of view; it is the sensation of green conveyed by a certain nerve-fibre.

These two things, which materially are so different, I replace schematically by a single thing which I call a point, and I express my thought by saying that in the one case as in the other, a point of the body has remained fixed. Thus every one of our new elements will be what is conserved by all the displacements of a sub-group; to every sub-group there will then correspond an element and *vice versa*.

Let us consider the different transformations of the same sub-group. They are infinite in number and may form a simple,

double, triple, continuous infinity. To each one of these transformations an element can be made to correspond; I have then a simple, double, triple, etc., infinity of them, and the degree of our continuous group is 1, 2, 3,

Suppose that we choose the different transformations of a rotative sub-group. We have here a triple infinity. The material of our group is accordingly composed of a triple infinity of elements. The degree of the group is three. We have then chosen the point as the element of space and given to space three dimensions.

Suppose we choose the different transformations of a helicoidal sub-group. Here we have a quadruple infinity. The material of our group is composed of a quadruple infinity of elements. Its degree is four. We then have chosen the straight line as the element of space,—which would give to space four dimensions.

Suppose, finally, that we choose the different transformations of a rotative sheaf. The degree would then be five. We have chosen as the element of space the figure formed by a straight line and a point on that straight line. Space would have five dimensions.

Here are three solutions, which are each logically possible. We prefer the first because it is the simplest, and it is the simplest because it is that which gives to space the smallest number of dimensions. But there is another reason which recommends this choice. The rotative sub-group first attracts our attention because it conserves certain sensations. The helicoidal sub-group is known to us only later and more indirectly. The rotative sheaf on the other hand is itself merely a sub-group of the rotative sub-group.

THE NOTION OF POINT.

I feel that I am here touching on the most delicate spot of this discussion, and I am compelled to stop for a moment to justify more completely my previous assertions which some persons may be disposed to doubt. Many persons, indeed, would consider the notion of a point of space as so immediate and so clear that any definition of it is superfluous. But I believe it will be granted me that so subtle a notion as that of the mathematical point, without

length, breadth, or thickness, is not immediate, and that it needs to be explained.

But is it the same with the vaguer and less precisely defined, yet more empirical notion, of *place*? Is there any one who does not fancy he knows perfectly well what he is talking about when he says: this object occupies the place which was just occupied by that object. To determine the range of such an assertion, and the conclusions which can be drawn from it, let us seek to analyse its signification. If I have moved neither my body, my head, nor my eye, and if the image of the object *B* affects the same retinal fibres that the image of the object *A* previously affected; if again, although I have moved neither my arm nor my hand, the same sensory fibres which extend to the end of the finger, and which formerly conveyed to me the impression which I attributed to the object *A* now convey to me the impression which I attribute to the object *B*; if both these conditions are fulfilled,—then ordinarily we agree to say that the object *B* occupies the place which previously the object *A* occupied.

Before analysing so complicated a convention as that just stated I shall first make a remark. I have just enunciated two conditions: one relating to sight, and one relating to touch. The first is necessary but not sufficient, for we say in ordinary language that the point on the retina where an image is formed gives us knowledge only of the direction of the visual ray, but that the distance from the eye remains unknown. The second condition is at once necessary and sufficient, because we assume that the action of touch is not exercised at a distance, and that the object *A* like the object *B* cannot act upon the finger except by immediate contact. All this agrees with what experience has taught us; namely, that the first condition can be fulfilled without the second being realised, but that the second cannot be fulfilled without the first. Let it be remarked that we have here something which we could not know *a priori*, that experience alone is able to demonstrate it to us.

Nor is this all. To determine the place of an object I made use only of an eye and a finger. I could have made use of several other means,—for example, of all my other fingers. Having been

made aware that the object *A* has produced upon my first finger a tactual impression, suppose that by a series of movements *S* my second finger comes into contact with the same object *A*. My first tactual impression ceases and is replaced by another tactual impression which is conveyed to me by the nerve of the second finger, and which I still attribute to the action of the object *A*. Some time afterwards, and without my having moved my hand, the same nerve of the second finger conveys to me another tactual impression, which I attribute to the action of another object *B*. I then say that the object *B* has taken the place of the object *A*.

At this moment I make a series of movements *S'* the inverse of the series *S*. How do I know that these two series are inverse to one another? Because experience has taught me that when the internal change *S* that corresponds to certain muscular sensations is followed by an internal change *S'* which corresponds to other muscular sensations, a compensation is effected and my primitive impressions, originally modified by the change *S*, are reestablished by the change *S'*.

I execute the series of movements *S'*. The effect ought to be to take back my first finger to its initial position and so to put it into contact with the object *B*, which has taken the place of the object *A*. I ought, therefore, to expect that the nerve of my first finger should convey to me a tactual sensation attributable to the object *B*. In fact this is what happens.

But would it therefore be absurd to suppose the contrary? And why would it be absurd? Shall I say that the object *B* having taken the place of the object *A*, and my first finger having resumed its original place, it ought to touch the object *B* just as before it touched the object *A*? This would be an outright begging of the question. And to show this let us attempt to apply the same reasoning to another example, or rather let us return to the example of sight and touch which I cited at the outset.

The image of the object *A* has made an impression on one of my retinal fibres. At the same time the nerve of one of my fingers conveys to me a tactual impression which I attribute to the same object. I move neither my eye nor my hand. And a moment after

the image of the object *B* has impressed the same retinal fibre. By a course of reasoning perfectly similar to that which precedes, I should be tempted to conclude that the object *B* had taken the place of the object *A*, and I should expect that the nerve of my finger would convey to me a tactual impression attributable to *B*. And yet I should be deceived. For the image of *B* may chance to be formed upon the same point of the retina as the image of *A*, although the distance to the eye may not be the same in the two cases.

Experience has refuted my reasoning. I extricate myself by saying that it is not sufficient for two bodies to cast their image upon the same retinal fibre in order to justify me in saying that the two bodies are in the same place; and I should extricate myself in a similar manner in the case of the two fingers, if the indications of the second finger had not been in accord with those of the first, and if experience had been at variance with my reasoning. I should still say that two objects *A* and *B* can make an impression upon the same finger by means of touch and yet not be in the same place; in other words, I should conclude that touch could be effected at a distance. Or, again, I should agree to consider *A* and *B* as being in the same place only on the condition of there being concordance not only between their effects upon the first finger, but also between their effects upon the second finger. One might almost say, in a certain point of view, that one more dimension would be attributed to space in this manner.

To sum up, there are certain laws of *concordance*, which can be revealed to us only by experience, and which are at the basis of the vague notion of place.

But even taking these laws of concordance for granted, can we deduce from them the much more precise notion of point and the notion of number of dimensions? This remains to be examined.

First an observation. We have spoken of two objects *A* and *B*, which have cast one after another their image on the same point of the retina. But these two images are not identical; otherwise how could I distinguish them? They differ, for example, in color. The one is red, the other is green. We have, accordingly, two sen-

sations which differ in quality and which are doubtless conveyed to me by two different though contiguous nerve-fibres. What have they in common with one another, and why am I led to associate them together? I believe that if the eye were immovable, we should never have thought of this association. It is the movements of the eye that have taught us that there is the same relation on the one hand between the sensation of green at the point *A* of the retina and the sensation of green at the point *B* of the retina, and on the other hand between the sensation of red at the point *A* of the retina and the sensation of red at the point *B* of the retina. We have found, in fact, that the same movements, corresponding to the same muscular sensations, cause us to pass from the first to the second, or from the third to the fourth. Were this not so, these four sensations would appear qualitatively distinct, and we should no more think of establishing a sort of proportion between them than we should between an olfactory, a gustatory, an auditive and a tactual sensation.

Yet whatever be the origin of this association, it is implied in the notion of place, which could not have grown up without it. Let us analyse, therefore, its laws. We can only conceive them under two different forms equally remote from mathematical continuity; namely, under the form of discontinuity or under the form of physical continuity.

Under the first form, our sensations will be divided into a very large number of "families"; all the sensations of one family being associated with one another and not being associated with those of other families. Since to every family there would correspond a place, we should have a finite but very large number of places, and the places would form a discrete aggregate. There would be no reason for classifying them in a table of three dimensions rather than in one of two or four; and we could not deduce from them either the mathematical point or space.

Under the second form, which is more satisfactory, the different families interpenetrate one another. *A*, for example, will be associated with *B*, and *B* with *C*. But *A* will not appear to us as associated with *C*. We shall find that *A* and *C* do not belong to

the same family, although on the one hand A and B , and on the other hand B and C , will appear to us as belonging to the same family. Thus we cannot distinguish between a weight of nine grammes and one of ten grammes, or between the latter weight and a weight of eleven grammes. But we can readily tell the difference between the first weight and the third. This is always the formula of physical continuity.

Let us picture to ourselves a series of wafers partially covering one another in such wise that the plane is totally covered; or better, let us picture to ourselves something analogous in a space of three dimensions. If these wafers were to form by their superposition only a sort of one-dimensional ribbon, we should recognise it, because the associations of which I have just been speaking obey a law that may be stated as follows: if A is associated at once with B , C , and D , D is associated with B or with C . This law would not be true if our wafers covered by their superposition a plane or a space of more than two dimensions. When I say, therefore, that all possible places constitute an aggregate of one dimension or of more than one dimension, I mean to say simply that this law is true or that it is false. When I say that they constitute an aggregate of two or three dimensions, I simply affirm that certain analogous laws are true.

Such are the foundations on which we may attempt to construct a *static* theory of the number of dimensions. It will be seen how complicated is this manner of defining the number of dimensions, how imperfect it is, and it is useless to remark upon the distance which still separates the physical continuity of three dimensions as thus understood from the real mathematical continuity of three dimensions.

DISCUSSION OF THE PRECEDING THEORY.

Without dwelling upon the multitude of difficult details, let us see in what those associations consist upon which the notion of place rests. We shall see that we are finally led back, after a long detour, to the notion of group, which appeared to us at the outset

the best fitted for elucidating the question of the number of dimensions.

By what means are different "places" distinguished from one another? How, for example, are two places occupied successively by the extremity of one of my fingers to be distinguished? Evidently by the movement which my body has made in the interval, movements which are made known to me by a certain series of muscular sensations. These two places correspond to two distinct attitudes and positions of the body which are known solely by the movements which I have had to make in changing a certain initial attitude and a certain initial position; and these movements themselves are known to me only by the muscular sensations which they have provoked.

Two attitudes of the body, or two corresponding places of the finger, appear to me identical if the two movements which I must make to reach them differ so little from each other that I cannot distinguish the corresponding muscular sensations. They will appear to me non-identical, without some new convention, if they correspond to two series of distinguishable muscular sensations.

But in this manner we have engendered not a physical continuity of three dimensions but a physical continuity of a much larger number of dimensions; for I can cause the muscular sensations corresponding to a very large number of muscles to vary, and I do not on the other hand consider a single muscular sensation only, nor even an aggregate of simultaneous sensations, but a series of successive sensations, and I can make the laws by which these sensations succeed one another vary in an arbitrary manner.

Why is the number of dimensions reduced, or, what is the same thing, why do we consider two places as identical when the two corresponding attitudes of the body are different? Why do we say in certain cases that the place occupied by the extremity of a finger has not changed, although the attitude of the body has changed?

It is because we discover that very *frequently*, in the movement which causes the passage from the one to the other of these two attitudes, the tactual sensation attributable to the contact of this finger with an object *A* persists and remains constant. We agree

then, to say that these two attitudes shall be placed in the same class and that this class shall embrace all attitudes corresponding to the same place occupied by the same finger. We agree that these two attitudes shall still be placed in the same class even when they are accompanied by no tactual sensation, or by variable tactual sensations.

This convention has been evoked by experience, because experience alone informs us that certain tactual sensations are frequently persistent. But in order that conventions of this kind shall be permissible, they must satisfy certain conditions which it now remains for us to analyse.

If I place the attitudes A and B in the same class, and also the attitudes B and C in the same class, it follows necessarily that the attitudes A and C must be regarded as belonging to the same class. If, then, we agree to say that the movements which cause the passage from the attitude A to the attitude B do not change the place of the finger, and if the same holds true of the movements which cause the passage from the attitude B to the attitude C , it follows necessarily that the same must again be true of those which cause the passage from the attitude A to the attitude C . In other words, the aggregate of the movements causing a passage from one attitude to another attitude of the same class constitutes a group. It is only when such a group exists that the convention above laid down is acceptable. To every class of attitudes, and consequently to every place, there will therefore correspond a group, and we are here led back again to the notion of group, without which there would be no geometry.

Nevertheless, there is a difference between the principle here under discussion and the theory which I developed above. Here each place appears to me associated with a certain group which is introduced as the sub-group S of the group G formed by the movements which can give to the body all possible positions and all possible attitudes, the relative situations of the different parts of the body being allowed to vary in any manner whatsoever. In our other theory, on the contrary, every point was associated with a sub-group S' of the group G' formed by the displacements of the

body viewed as an invariable solid, that is to say, by displacements such that the relative situations of the different parts of the body do not vary.

Which of the two theories is to be preferred? It is evident that G' is a sub-group of G and S' a sub-group of S . Further, G' is much simpler than G , and for this reason the theory which I first propounded and which is based upon the consideration of the group G' appears to me simpler and more natural, and consequently I shall hold to it.

But be this as it may, the introduction of a group, more or less complicated, appears to be absolutely necessary. Every purely statical theory of the number of dimensions will give rise to many difficulties, and it will always be necessary to fall back upon a dynamical theory. I am happy to be in accord on this point with the ideas set forth by Professor Newcomb in his *Philosophy of Hyper-space*.

THE REASONING OF EUCLID.

But in order to show that the idea of displacement, and consequently the idea of group, has played a preponderant part in the genesis of geometry, it remains to be shown that this idea dominates all the reasoning of Euclid and of the authors who after him have written upon elementary geometry.

Euclid begins by enunciating a certain number of axioms; but it must not be imagined that the axioms which he enunciates explicitly are the only ones to which he appeals. If we carefully analyse his demonstrations we shall find in them, in a more or less masked form, a certain number of hypotheses which are in reality axioms disguised; and we may say almost as much of some of his definitions.

His geometry begins with declaring that two figures are equal if they are superposable. This assumes that they can be displaced and also that among all the changes which they may undergo, we can distinguish those which may be regarded as displacements without deformation. Again, this definition implies that two figures which are equal to a third are equal to each other. And that

is tantamount to saying that if there be a displacement which puts the figure *A* upon the figure *B*, and a second displacement which superposes the figure *B* upon the figure *C*, there will also be a third, the resultant of the first two, which will superpose the figure *A* upon the figure *C*. In other words, it is presupposed that the displacements form a group. The notion of a group, accordingly, is introduced from the outset, and inevitably introduced.

When I pronounce the word "length," a word which we frequently do not think necessary to define, I implicitly assume that the figure formed by two points is not always superposable upon that which is formed by two other points; for otherwise any two lengths whatever would be equal to each other. Now this is an important property of our group.

I implicitly enunciate a similar hypothesis when I pronounce the word "angle."

And how do we proceed in our reasonings? By displacing our figures and causing them to execute certain movements. I wish to show that at a given point in a straight line a perpendicular can always be erected, and to accomplish this I conceive a movable straight line turning about the point in question. But I presuppose here that the movement of this straight line is possible, that it is continuous, and that in so turning it can pass from the position in which it is lying on the given straight line, to the opposite position in which it is lying on its prolongation. Here again is a hypothesis touching the properties of the group.

To demonstrate the cases of the equality of triangles, the figures are displaced so as to be superposed one upon the other.

Finally, what is the method employed in demonstrating that from a given point one and only one perpendicular can always be drawn to a given straight line? The figure is turned 180° around the given straight line, and in this manner the point symmetrical to the given point with respect to the given straight line is obtained. We have here a feature most characteristic, and here appears the part which the straight line most frequently plays in geometrical demonstrations, namely, that of an axis of rotation.

There is implied here the existence of the sub-group which I

have called the rotative sheaf. When—which also frequently happens—a straight line is made to slide along itself (for we shall, of course, continue to suppose that it can serve as an axis of rotation), we implicitly take the existence of the helicoidal sub-group for granted. In fine, the principal foundation of Euclid's demonstrations is really the existence of the group and its properties.

Unquestionably he appeals to other axioms which it is more difficult to refer to the notion of group. An axiom of this kind is that which some geometers employ when they define a straight line as the shortest distance between two points. *But it is precisely such axioms that Euclid enunciates.* The others, which are more directly associated with the idea of displacement and with the idea of groups, are the very ones which he implicitly admits, and which he does not deem it even necessary to enunciate. This is tantamount to saying that the former are the fruit of a later experience, that the others were first assimilated by us, and that consequently the notion of group existed prior to all the others.

THE GEOMETRY OF STAUDT.

It is known that Staudt attempted to base geometry upon different principles. Staudt admits the following axioms only:

1. Through two points a straight line can always be drawn.
2. Through three points a plane can always be drawn.
3. Every straight line which has two of its points in a plane lies entirely in that plane.
4. If three planes have one point in common, and one only, any straight line will cut at least one of these three planes.

These axioms are sufficient to establish all the *descriptive* properties relating to the intersections of straight lines and planes. To obtain the metrical properties we begin with *defining* a harmonic pencil of four straight lines, taking as definition the well-known descriptive property. Then the anharmonic ratio of four points is *defined*, and finally, supposing that one of these four points has been relegated to infinity, the ratio of two lengths is *defined*.

This last is the weak point of the foregoing theory, attractive though it be. To arrive at the notion of length by regarding it

merely as a particular case of the anharmonic ratio is an artificial and repugnant detour. This evidently is not the manner in which our geometrical notions were formed.

Let us see now whether we can conceive, without the introduction of the notion of group and of movement, how the notions which serve as the foundations of this ingenious geometry have taken their rise. Let us see what experiences might have led us to formulate the axioms enunciated above.

If the straight line is not given as an axis of rotation, it can be given only in one way, namely, as the trajectory of a ray of light. I mean, that the experiences, always more or less crude, which serve us as our point of departure, should all be applicable to the ray of light, and that we must define the straight line as a line for which the simple laws which the ray of light approximately obeys will be rigorously true. The following is the experience which must be made in order to verify the most important of our axioms, namely, the third.

Let two threads be stretched. Let the eye be placed at the extremity of one of these threads. We see that the thread is entirely hidden by its extremity, which teaches us that the thread is rectilinear, that is to say, is the direction of the trajectory of a ray of light. Let the same be done for the second thread. The following is then observed: either there will be no position of the eye for which one of the threads is entirely hidden by the other, or there will be an infinity of such positions.

How is the question of the number of dimensions presented in this order of ideas? Let us consider all the positions of the eye for which one of the strings is hidden by the other. Let us suppose that in one of these positions the point A of the first string is hidden by the point A' of the second, the point B by the point B' , the point C by the point C' . We then discover that if the body is so displaced that the point A is always hidden by the point A' and the point B by the point B' , that the point C always remains hidden by the point C' , and that in general any point whatsoever of the first thread remains hidden by the same point of the second thread by which it was hidden before the body was displaced. We ex-

press this fact by saying that although the body is displaced, the position of the eye has not changed.

We see thus that the position of the eye is defined by two conditions, viz., that A is hidden by A' and B by B' . We express this fact by saying that the *locus* of the points such that the two threads mutually hide each other has two dimensions.

Similarly, let us suppose that in a certain position of the body four threads A, B, C, D , hide four points A', B', C', D' ; let us suppose that the body is displaced, but in such a manner that A, B , and C continue to hide A', B' , and C' . We shall then discover that D continues to hide D' , and we shall again express this fact by saying that the position of the eye has not changed. This position will therefore be defined by three conditions, and this is why we say that space has three dimensions.

It will be remarked that the law as thus experimentally discovered, is only approximately true. But this is not all. It is not even always true, because D or D' may have moved at the same time that my body was being displaced. We then simply declare that this law is often approximately true.

But we are desirous of arriving at geometrical axioms which are rigorously and always true, and we always escape the dilemma by the same artifice, namely, by saying that we agree to consider the change observed as the resultant of two others, viz., of one which rigorously obeys the law and which we attribute to the displacement of the eye, and of a second one which is generally very small and which we attribute either to qualitative alterations or to the movements of external bodies.

We have not been able to avoid the consideration of movements of the eye and of the body, yet we may say, that from a certain point of view the geometry of Staudt is predominantly a visual geometry, while that of Euclid is predominantly muscular.

Undoubtedly unconscious experiences analogous to those of which I have just spoken may have played a part in the genesis of geometry; but they are not sufficient. If we had proceeded, as the geometry of Staudt supposes us to have done, some Apollonius would have discovered the properties of polars. But it would have

been only long after, that the progress of science would have made clear what a length or an angle is. We should have had to wait for some Newton to discover the various cases of the equality of triangles. And this is evidently not the way that things have come to pass.

THE AXIOM OF LIE.

It is Sophus Lie who has contributed most towards making prominent the importance of the notion of group and laying the foundations of the theory that I have just expounded. It is he, in fact, who gave the present form to the mathematical theory of continuous groups. But to render possible its application to geometry, he regards a new axiom as necessary, which he enunciates by declaring that space is a *Zahlenmannigfaltigkeit*; that is, that to every point of a straight line there corresponds a number and *vice versa*.

Is this axiom absolutely necessary? And could not the other principles which Lie has laid down dispense with it? We have seen above in connexion with continuity, that the best known groups may be distributed from a certain point of view into three classes; all the operations of the group can be divided into sheaves; for "discontinuous" groups the different operations of the same sheaf are only a single operation repeated once, twice, three times, etc.; for "continuous" groups properly so called the different operations of the same sheaf correspond to different whole numbers, commensurable or incommensurable; finally, for groups which may be called "semi-continuous," these operations correspond to different commensurable numbers.

Now it may be demonstrated that no discontinuous or semi-continuous group exists possessing other properties than those which experience has led us to adopt for the fundamental group of geometry, and which I here briefly recall: The group contains an infinity of sub-groups, all *gleichberechtigt*, which I call rotative sub-groups. Two rotative sub-groups have a sheaf in common which I call rotative and which is common not only to two but also to an infinity of rotative sub-groups. Finally, every very small displace-

ment of the group may be regarded as the resultant of six displacements belonging to six given rotative sheaves. A group satisfying these conditions can be neither discontinuous nor semi-continuous.

Unquestionably this is an exceedingly recondite property, and not easy to demonstrate. Geometers who were ignorant of it have not the less hit upon its consequences, as for example, when they learned that the ratio of a diagonal to the side of a square is incommensurable. It was for this reason that the introduction of incommensurables into geometry became necessary.

The group, therefore, must be continuous, and it seems as if the axiom of Lie were useless.

Nevertheless, we are obliged to remark that the classification of groups above sketched is not complete; groups may be conceived which are not included in it. We might, therefore, suppose that the group is neither discontinuous, semi-continuous, nor continuous. But this would be a complex hypothesis. We reject it, or rather we never think of it, for the reason that it is not the simplest compatible with the axioms adopted.

The foundation of the axiom of Lie remains to be supplied.

GEOMETRY AND CONTRADICTION.

In following up all the consequences of the different geometrical axioms, are we never led to contradictions? The axioms are not analytical judgments *a priori*; they are conventions. Is it certain that all these conventions are compatible?

These conventions, it is true, have all been suggested to us by experiences, but by crude experiences. We discover that certain laws are approximately verified, and we decompose the observed phenomenon conventionally into two others: a purely geometrical phenomenon which exactly obeys these laws; and a very minute disturbing phenomenon.

Is it certain that this decomposition is always permissible? It is certain that these laws are *approximately* compatible, for experience shows that they are all approximately realised at one and the same time in nature. But is it certain that they would be compatible if they were absolutely rigorous?

For us the question is no longer doubtful. Analytical geometry has been securely established, and *all* the axioms have been introduced into the equations which serve as its point of departure; we could not have written these equations if the axioms had been contradictory. Now that the equations are written, they can be combined in all possible manners; analysis is the guarantee that contradictions shall not be introduced.

But Euclid did not know analytical geometry, and yet he never doubted for a moment that his axioms were compatible. Whence came his confidence? Was he the dupe of an illusion? And did he attribute to our unconscious experiences more value than they really possess? Or perhaps, since the idea of the group was potentially pre-existent in him, did he have some obscure instinct for it, without reaching a distinct notion of it? I shall leave the question undecided although inclined towards the second solution.

THE USE OF FIGURES.

It may be asked why geometry cannot be studied without figures. This is easy to account for. When we commence studying geometry, we have already had in innumerable instances the fundamental experiences which have enabled our notion of space to originate. But they were made without method, without scientific attention and unconsciously, so to speak. We have acquired the ability *to represent to ourselves* familiar geometrical experiences without being obliged to resort to material reproductions of them; but we have not yet deduced from them logical conclusions. How is this to be done? Before enunciating the law, the experience in question is perceptually represented by stripping it as completely as possible of all accessory or disturbing circumstances,—just as a physicist eliminates the sources of systematic error in his experiments. It is here that figures are necessary, but they are an instrument only slightly less crude than the chalk which is employed in drawing them; and, like material objects, it is beyond our power to represent them in the geometrical space which forms the object of our studies; we can only represent them in sensible space. We

accordingly do not study material figures, but simply make use of them in studying something which is higher and more subtle.

FORM AND MATTER.

We owe the theory which I have just sketched to Helmholtz and Lie. I differ from them in one point only, but probably the difference is in the mode of expression only and at bottom we are completely in accord.

As I explained above, we must distinguish in a group the form and the matter [material]. For Helmholtz and Lie the matter of the group existed previously to the form, and in geometry the matter is a *Zahlenmannigfaltigkeit* of three dimensions. *The number of dimensions is therefore posited prior to the group.* For me, on the contrary, the form exists before the matter. The different ways in which a cube can be superposed upon itself, and the different ways in which the roots of a certain equation may be interchanged, constitute two isomorphic groups. They differ in matter only. The mathematician should regard this difference as superficial, and he should no more distinguish between these two groups than he should between a cube of glass and a cube of metal. In this view the group exists prior to the number of dimensions.

We escape in this way also an objection which has often been made to Helmholtz and Lie. "But your group," say these critics, "presupposes space; to construct it you are obliged to assume a continuum of three dimensions. You proceed as if you already knew analytical geometry." Perhaps the objection was not altogether just; the continuum of three dimensions which Helmholtz and Lie posited was a sort of non-measurable magnitude analogous to magnitudes concerning which we may say that they have grown larger or smaller, but not that they have become twice or three times as large.

It is only by the introduction of the group, that they made of it a measurable magnitude, that is to say a veritable space. Again, the origin of this non-measurable continuum of three dimensions remains imperfectly explained.

But, it will be said, in order to study a group even in its formal properties, it is necessary to construct it, and it cannot be constructed without matter. One might as well say that one cannot study the geometrical properties of a cube without supposing this cube to be of wood or of iron. The complexus of our sensations has without doubt furnished us with a sort of matter, but there is a striking contrast between the grossness of this matter and the subtle precision of the form of our group. It is impossible that this can be, properly speaking, the matter of such a group. The group of displacements such as it is given us directly by experience, is something more gross in character; it is, we may say, to continuous groups proper what the physical continuum is to the mathematical continuum. We first study its form agreeably to the formula of the physical continuum, and since there is something repugnant to our reason in this formula we reject it and substitute for it that of the continuous group which, potentially, pre-exists in us, but which we originally know only by its form. The gross matter which is furnished us by our sensations was but a crutch for our infirmity, and served only to force us to fix our attention upon the pure idea which we bore about in ourselves previously.

CONCLUSIONS.

Geometry is not an experimental science; experience forms merely the occasion for our reflecting upon the geometrical ideas which pre-exist in us. But the occasion is necessary; if it did not exist we should not reflect; and if our experiences were different, doubtless our reflexions would also be different. Space is not a form of our sensibility; it is an instrument which serves us not to represent things to ourselves, but to reason upon things.

What we call geometry is nothing but the study of formal properties of a certain continuous group; so that we may say, space is a group. The notion of this continuous group exists in our mind prior to all experience; but the assertion is no less true of the notion of many other continuous groups; for example, that which corresponds to the geometry of Lobatchévski. There are,

accordingly, several geometries possible, and it remains to be seen how a choice is made between them. Among the continuous mathematical groups which our mind can construct, we choose that which deviates least from that rough group, analogous to the physical continuum, which experience has brought to our knowledge as the group of displacements.

Our choice is therefore not imposed by experience. It is simply guided by experience. But it remains free; we choose this geometry rather than that geometry, not because it is more *true*, but because it is the more *convenient*.

To ask whether the geometry of Euclid is true and that of Lobatchévski is false, is as absurd as to ask whether the metric system is true and that of the yard, foot, and inch, is false. Transported to another world we might undoubtedly have a different geometry, not because our geometry would have ceased to be true, but because it would have become less convenient than another. Have we the right to say that the choice between geometries is imposed by reason, and, for example, that the Euclidean geometry is alone true because the principle of the relativity of magnitudes is inevitably imposed upon our mind? It is absurd, they say, to suppose a length can be equal to an abstract number. But why? Why is it absurd for a length and not absurd for an angle? There is but one answer possible. It appears to us absurd, because it is contrary to our habitual way of thinking. Unquestionably reason has its preferences, but these preferences have not this imperative character. It has its preferences for the simplest because, all other things being equal, the simplest is the most convenient. Thus our experiences would be equally compatible with the geometry of Euclid and with a geometry of Lobatchévski which supposed the curvature of space to be very small. We choose the geometry of Euclid because it is the simplest. If our experiences should be considerably different, the geometry of Euclid would no longer suffice to represent them conveniently, and we should choose a different geometry.

Let it not be said that the reason why we deem the group of Euclid the simplest is because it conforms best to some pre-existing ideal which has already a geometrical character; it is simpler be-

cause certain of its displacements are interchangeable with one another, which is not true of the corresponding displacements of the group of Lobatchévski. Translated into analytical language, this means that there are fewer terms in the equations, and it is clear that an algebraist who did not know what space or a straight line was would nevertheless look upon this as a condition of simplicity.

In fine, it is our mind that furnishes a category for nature. But this category is not a bed of Procrustes into which we violently force nature, mutilating her as our needs require. We offer to nature a choice of beds among which we choose the couch best suited to her stature.

H. POINCARÉ.

PARIS, FRANCE.

ON PASIGRAPHY.

ITS PRESENT STATE AND THE PASIGRAPHIC MOVEMENT IN ITALY.

THE following communication was delivered in the German language at Zürich in August 1897 before the first International Congress of Mathematicians, first Section (for Arithmetic and Algebra). The Congress, which was attended by over 240 persons from nearly every civilised part of the globe, proved to be a remarkable success, though, owing to the fact that most of the British and American mathematicians were on their way to the Meeting of the British Association at Toronto, the English-speaking element was but scantily represented, there being only ten such persons present at the most. The next congress is to take place at Paris in 1900. The idea of starting such a congress having already been mooted at the meeting of the German Society of naturalists and physicians at Frankfurt a. M. in 1896, it ripened into a workable shape. There the opinion prevailed that the English language, being neutral ground between the French and the German, would be elected as the official means of communication, agreeably to which opinion the author had prepared his paper in English.¹ We are glad to put the original—since but slightly altered—before our readers nearly at the same time that the Reports of the Congress appear.

¹ The editors have been careful to preserve all the stylistic and typographical details of the original MS. of Professor Schröder.—*Ed.*

At an *international* Congress of Mathematicians there is in my opinion scarcely any topic more worthy of discussion, than that of Pasigraphy. For the aim of this novel branch of Science is nothing less than the ultimate establishment of a scientific Language, entirely free from national peculiarities, and through its very construction conveying the foundation of exact and true philosophy.

Such a language of course cannot be created at once for the whole realm of human thought. Its most important and hitherto mainly realised parts appear to be those which concern the *fundamental notions of pure Mathematics*, especially *Logic, Arithmetic, Geometry*.

I shall chiefly confine myself to some of these departments.

Time will not permit me to enter into an historical exposition. Suffice it to bring to recollection, that the pasigraphic discipline was clearly foreseen and postulated by DESCARTES, and that it formed an ideal hovering before the mind of LEIBNIZ during his whole life. As my accomplished friend Signor PEANO has recently pointed out, Leibniz so much cherished and appreciated the idea, that he says: except the founder of a religion or the ruler of a state—*praeter Prophetam ac Principem*—no person could better serve humanity than he who would realise that ideal—then so far away and actually not much more than a dim concept.

Leibniz also complained of the very small interest his contemporaries exhibited in the matter. The same complaint would in most quarters prove just as well founded now-a-days. However I venture to trust, that on the present occasion I may be fortunate enough to arouse some enthusiasm for this very important subject, which *now* appears to have entered upon a very promising stage.

Still at the outset it is necessary to contradict Signor Peano's statement of 1894 in his "*Introduction au formulaire de mathématique*," p. 52, that: "*Le problème proposé par Leibniz est (donc) résolu.*" With this sanguine dictum he—as we shall see—altogether anticipated the actual and impending achievement of pasigraphic science. For when his assertion was uttered, not even the indispensable means for attaining the goal had then been secured

or rendered generally accessible, as they are at present. But even at this date there is yet much hard work to be gone through.

The problem to be solved for any given branch of science amounts to: expressing *all* the notions which it comprises, adequately and in the concisest possible way, through a minimum of *primitive notions*, say "categories," by means of purely logical operations of general applicability, thus remaining the same for every branch of science and being subject to the laws of ordinary Logic, but which latter will present themselves in the shape of a "calculus ratiocinator." For the categories and the operations of this "lingua characteristica" or "scriptura universalis" easy signs and simple symbols, such as letters, are to be employed, and—unlike the "words" of common language—they are to be used with absolute consistency (with perfect "Konsequenz," as we Germans say, or mathematical strictness, "Strenge").

It is almost superfluous to emphasise on how much higher a level this, our logical, aim stands, as compared with the merely linguistic endeavors of the Volapükists for instance, who are only striving to create means of mutual comprehension among the users of different languages, and the very mention of whom nearly amounts to a degradation of our object.

It may once for all be explicitly stated, that the pasigraphic language is not in the least destined ever to be *spoken*, but only to serve and forward on account of its logical structure the purposes of Science; first of all of that science, which the ancient Greek called "*the science* (katexochen)," Mathesis, and next: of Logic and an exact Philosophy, so long lacking, and hence to be hoped for—at last!

As an individual opinion of mine, perhaps not as yet shared by many, I may be permitted to state, by the way, that I consider pure Mathematics to be only one branch of general Logic, the branch originating from the creation of Number, to the economical virtues of which is due the enormous development that particular branch has been favored with in comparison with the other branches of Logic that until of late almost remained stationary. This view is confirmed by the fact, that under the pasigraphic aspect Arith-

metic can do without any peculiar categories or primitive notions—those of general logic sufficing to compose all its notions (such as multitude, number, finiteness, limes, function, Abbildung or one-to-one correspondence, addition, etc.).

If we limit our considerations to purest Mathematics it is indeed already becoming manifest—chiefly owing to the development which Charles S. PEIRCE's Logic of Relatives has experienced—that all its notions, as well as those of Logic in general, are reducible to only *five* primitive notions or categories in the Aristotelian and Kantian sense.

Before these are presented a remark is needed.

The minimum number of indispensable symbols will exceed the said number 5 of the categories, some of which must find a representation by symbols doubly: just as in arithmetic neither of the two signs

$$+ \text{ and } \Sigma$$

can in the long run or eventually be dispensed with, notwithstanding that both merely serve to represent the unique notion of an arithmetical *sum*.

Besides, these categories do not constitute the whole of the system of fundamental denotations. Whereas for instance parentheses or brackets form a very important and practically indispensable element of denotation, yet do not represent any notion at all and in themselves are devoid of meaning. (As is well known, brackets only serve in our symbolic language, as in Algebra, to characterise any compound of symbols, when included by them, as forming *one name*.) Moreover, we are constantly led to employ letters in the quality of *general* symbols, ready for such use, because of their having no fixed meaning attached to them.

This settled, the 5 categories or primitive notions of general logic with the inclusion of arithmetic are those which form the upper line in the following set:

$$1) \left\{ \begin{array}{|c|c|c|c|c|} \hline \equiv & \cdot & - & \sim & ; \\ \hline 1 & \Pi & & & \end{array} \right\}$$

—the two first ones, as may be seen, being doubly represented, the

second one even trebly, whereas the multiplication point (between letters) may as well be omitted, the result being a juxtaposition.

The first, being the well-known sign of equality, is in general logic to be interpreted in a much more restricted sense than in mathematics, viz., to mean *identity* or sameness; its equivalent 1' (a "one" with an apostrophe,—I pronounce it for shortness: one-ap), puts forth the same category of identity as a relative term, destined to represent the class of things that are "equal to-" or "identical with-", which sign sometimes also may be translated simply by the word "itself" = selbst = le même = lo mismo.

The second or multiplication point is used in general logic—wholly independently of its arithmetical meaning—to express the category of *intersection*, Schnitt, since its office is always to denote that which is *common* to both the terms joined (and separated) by it. The *II* is then, analogically as in arithmetical analysis, employed for indicating "identical products" resulting from the operation of such intersection. $a \cdot b$ or ab means: what is (at once) *a* and *b*.

Our third category, to be represented by an *overstroke*, is the well-known logical operation of denial or *negation*. The sign—to speak more exactly—is intended to indicate its result, the negate. If *a* means anything, then \bar{a} denotes what is not-*a*. Evidently negation is a primitive notion or category, incapable of experiencing a formal definition. In lieu of the lacking definition, the so-called logical "principles" of contradiction and excluded middle step in to fill the gap. And by the bye be it said that similarly all principles of Logic as well as of Arithmetic would prove on examination to be mere substitutes for definitions (Peirce) and do not bear the character of axioms at all. (As is generally recognised, not every thing from the outset can be defined, since every definition has to rely on previous other notions, or categories already given.)

Our fourth category, represented by a crescent (to be placed over any letter), is that of conversion: if *a* means cause of-, then \bar{a} (*a*-converse) will denote effect of-, when *c* denotes child of, then \bar{c} is to denote parent (i. e., father or mother) of-. I purpose to return to this point.

The fifth category, which I represent by a *semicolon* (Strich-

punkt), is that of *relation* in general; the usual translation into words, of our semicolon being the particle "*of*," equalling "*von*," same as "*de*," the well-known predicate of nobility. If $a = \text{amans}$ means lover and b means benefactor, then $a;b$ denotes: lover of a benefactor. The operation consisting in the use of this sign is called relative multiplication or composition.

These five categories and their seven signs essentially suffice to embody all the fundamental notions of Logic and Arithmetic,—as will be seen afterwards: I shall have to justify this apparently very daring assertion at least to some extent here in detail.

But if theoretically they prove sufficient, in practice it will not answer to restrain ourselves to their exclusive use. In order to avoid extreme cumbrousness, to secure the benefit of terseness or brevity and to facilitate clear surveys, also out of regard to symmetry, we are compelled immediately to supplement the foregoing system of juxtaposed signs.

The following three lines show how the 18 symbols of the succeeding set, which are forming our complete system of denotation (in general Pasigraphy), reduce to our five categories.

11 supplementary definitions:

$$2) \begin{cases} 0 = a.\bar{a}, 1 = \bar{0}, 0' = \bar{1}, a+b = \bar{a}.\bar{b}, \Sigma a = \bar{\Pi a}, a;b = \bar{a};\bar{b}, \\ (a \Leftarrow b) = (a = a.b), (a \Leftarrow b) = \bar{a} \Leftarrow \bar{b}, (a < b) = (a \Leftarrow b).(b \Leftarrow a), \\ (a + b) = \bar{a} = \bar{b}, (a \Leftarrow b) = \bar{a} < \bar{b}. \end{cases}$$

The 18 signs:

$$3) 0, 1, +, \cdot, \Sigma, \Pi, 0', 1', -, \sim, (*), j, :, \Leftarrow, =, <, \Leftarrow, +, \Leftarrow.$$

Let us deal with these rapidly.

By the first of these equations is defined the logical notion of *Nothing*, which in general Logic is to be denoted by the cipher naught, 0. Whenever the need should arise to use the same sign for the *number* naught or zero, very much to be distinguished therefrom, I prevent their being confounded by putting a dot over the latter: $\dot{0}$. "*Nothing*" is here defined as that which is at once a and not- a , no matter what a may mean.

The next equation defines "*something*" as not-nothing. This notion comprises everything of which it is possible to speak, the Thinkable, and the sign 1 (one) thus is to represent in general

Logic the Totum, the notion of *All*, the Whole, der Denkbereich, say the "Universe of discourse." This also may occasionally be further restricted for the purpose of any special investigation. For preventing its sign from being confounded with the *number* one (1) as only can, and seldom will, occur in researches of a mixed character, both logical and arithmetical, it is my practice in the latter case to put a dot over it. (Similarly in such a case I employ the sign \times for indicating arithmetical multiplication and a larger $+$ for the arithmetical addition.)

The third equation 2) defines the relative term "different from" or "other than—" as being not identical with-, and introduces for designating it an apostrophised naught, to be spoken naught-ap. If this relation is to be stated *between* two terms, it is already customary (in German mathematical periodicals) to express it by a sign of equation cancelled by a down stroke and thus negated in effigy, \neq thus meaning unequal—see the definition last but one.

The fourth and fifth equation define the "identical *sum*" or logical aggregate (Inbegriff, Gesamtheit) to be denoted in general Logic by the signs borrowed from Arithmetic $+$ and Σ . $a+b$ is here to express that which is not at once not- a and not- b ; id est: what is either a or b , perhaps both.

The sixth equation 2) introduces a sign \ddagger "plus with a scorpion tail to the left" that I pronounce with the Italian word for +, viz., as *più*, for designating a relation very strange to ordinary thinking: a relation hitherto without a name. $a\ddagger b$ is to represent that which is not a not- a of a not- b , and this amounts to: an a at any rate of everything but b 's (no matter, whether it is an a of b 's also, or not). The operation of connecting a with b by means of this sign \ddagger , which thus results in the formation of the notion $a\ddagger b$, is called *relative addition*. The introduction of this apparently somewhat intricate unfamiliar notion is dictated by a regard for symmetry. In Logic whenever a class a is formed, the same interest as to the individuals *within* is due to those *without* this class, i. e., to the not- a . There is a duality of notion (dualism) between "containing" and "being contained in," \supset and \Leftarrow —see further on. Thus the \ddagger relation corresponds to the category of ($;$) exactly

in the same way as product and sum or the \cdot and $+$, as the particles *and* and *or* correspond with each other, surely none of which could be missed. To give an example at once, if t means Teiler, divisor of-, and if we restrain the Universe of discourse to the common numbers, then $t \vdash 0$ will express: what is a divisor of every number, save nothing or no number, and this simply means: a divisor of every number. Such indeed is the numerical unit, the number one, and none other.

Our next definition introduces the all-important notion of *implication* or *inclusion*, the being *contained in-* as a *part* anyhow (that is may-be as a *proper* or genuine part, echter Teil, or may even be as the whole itself). The implication or subsumption $a \Leftarrow b$, to be read as "*a* is contained within *b*," or "*a* is part of *b*," appears to be explained here by: *a* is identical with that, which is at once *a* and *b*. My implication sign \Leftarrow (in German to be read "eingeordnet") generally translates the copula "*is*," "*est*" of a categoric statement, and also, when placed between statements *a* and *b*, presents itself as the sign of inference or illation: for though the conclusion is in a certain sense *implied by* or *involved in* the premises, however *conversely*, if *b* follows from *a*, the class of occasions when *a* holds good, will be contained within the class of cases where *b* holds. The subsumption $a \Leftarrow b$ then may be read as: whenever *a* is true then *b* is true.

The next definition only introduces the denial of the foregoing relation: being not contained in-, (I need not enlarge thereon) in the same way as the last definition introduces the denial of the one we only have yet to discuss.

In the remaining definition 2), also an important one, is explained the relation of being contained in- as a *proper* part: *a* is contained in *b* as such, $a \subset b$, whenever *a* is contained in *b*, whilst *b* is not contained in *a* (or is other than *b*).

This again settled so far, we are in possession of and we command the complete denotation-system of general Logic, which consists of these eighteen signs 3), henceforth rendered legitimate for the use of Pasigraphy by their reduction to the five categories.

The system of denotation expounded is that which has natur-

ally arisen from the profound and persistent investigations, extending through nearly half a century, of men of the genius of DE MORGAN, BOOLE and most of all of one of the keenest American thinkers: Mr. CHARLES S. PEIRCE.¹ In working over his theory at large I have but slightly and never without intrinsic reasons, modified his (or Boole's) denotations, deviating only *slightly*, at least as compared with the divergencies shown by every system of denotation that derives from other sources, especially that of Signor PEANO and the Italian school. I shall call the former for simplicity's sake "Peirce's system" (omitting the addition "as modified by me"). For the benefit of those who are already familiar with the symbolism of the latter (Peano) and his most numerous, active and skilful adherents it may at once be stated that our signs

4) $\left\{ \begin{array}{l} 0, 1, +, \cdot, \Sigma, \Pi, \bar{a}, \Leftarrow \\ \text{correspond to the } \Lambda, V, \cup, \cap, \cup', \cap', -a, \epsilon, \supset \text{ of Peano.} \end{array} \right.$

By the way, since the signs Σ (and Π), as is well known, have to serve as the bearers, scaffolding, support, *frame* for the shifting suffix (Summationsvariable), which is to pass through a series of values, and besides for the limits (upper and under) of that sum or series, the substitutes above given by Peano for these Σ and Π appear to be chosen still less happily, than if in arithmetical analysis we should propose to replace the Σ , Π , generally in use, by a '+' and 'x'. Such an "emendation" turning out to be but a deterioration, would in German be ironically styled: "eine Verschlimmbesserung."

Touching the essential divergence, that Peano's denotation-system lacks our fifth category "of," the most important of all, and that in consequence it cannot show any signs corresponding to our relative operations (\dagger and $;$), I shall have a few words to say further on.

Now the *calculus ratiocinator* ruling, nay governing, our categories and fundamental operations, to the laws of which these primitive elements of thought are of necessity subject, is none other than Peirce's "*Algebra of Relatives*," a discipline (branch of science)

¹Not, however, infallible, as will be seen on a future occasion.

crowning the edifice of the "Algebra of Logic" and comprising as well the statement-calculus as the class-calculus—both as very subordinate parts.

Almost everything may be viewed as, or considered under the aspect of, a (dual or) *binary relative*, and can be represented as such. Even statements submit to be looked at and treated as binary relatives. Classes, assemblages (Mengen, ensembles) or absolute terms may be thus presented.

And since in ordinary as well as in scientific thinking the relative notions by far prevail over the absolute ones, which latter, over and above, are eventually comprised in and superseded by them, it is evident, that the Logic of the relative notions, Relatives, must form the indispensable base and underlie every successful attempt at Pasigraphy.

In the fact that traditional Logic so long confined itself to the absolute notions with the meagre categories of "all," "some," and "none" is to be perceived an essential cause for its stagnation, that undeniable standstill, which yet entitled KANT in his time to make the assertion: that during the two thousand years since ARISTOTLE Logic had not accomplished any real progress. This would now no longer accord with the facts.

And as for the present time, it may warningly be said that whosoever, while aiming at our logical ends, tries to erect the building on a narrower ground than that created and offered by the De Morgan-Peirce theory (which reposes on the general notions of relation, Relative and composition), such as for instance would be furnished by the introduction and admission among the categories merely of the notion of "function," or say of "transformation," or else of "(one to one) correspondence"—these altogether being by far more special, and only particular cases of that general notion of Relatives—whoever contents himself with logograms for any of such special notions will preclude himself from participating in and benefiting from the above already highly developed theory; he will bar, nay block, for himself the way to expeditious progress.

Let us now illustrate the scope and purport of our novel Logic of Relatives, and therewith demonstrate, at least to a large extent,

the sufficiency, here claimed, of the set of our five primitive notions 1) for building up the complete body of the fundamental notions of Arithmetic. I shall therefore put before the reader, arranged in several groups, the pasigraphic representation and definition of not a few of its most essential notions—to be aided by subsequent remarks.

- 5) $\left\{ \begin{array}{l} (a \text{ is a class, assemblage, collection, set, an absolute term, Ge-} \\ \text{biet, System, Menge, ensemble, insieme}) = (a; 1 = a) = \\ = 0 \ddagger \bar{a} \ddagger a \ddagger 0. \end{array} \right.$
- 6) $(\text{num. } a = 0) = (a = 0) = 0 \ddagger \bar{a} \ddagger 0.$
- 7) $\left\{ \begin{array}{l} (\text{num. } a = 1) = (a \text{ is an individual, element, constant function,} \\ \text{may be a single number}) = (0'; a; 1 = \bar{a}) = \\ = (0'; a = \bar{a}) = (a \nless 0'; a) = \bar{a}; (1' \ddagger \bar{a}). \end{array} \right.$
- 8) $\left\{ \begin{array}{l} (\text{num. } a = 2) = (\text{the set } a \text{ is a couple, pair}) = \\ = (0' a \bar{a} \nless 0'; a 0') = \bar{a}; 0' \{1' \ddagger (\bar{a} + 1')\}; a, \\ \text{which is contracted from} \\ \Sigma_y (i + j) (i + j \nless a) \Pi_k \{ (k + i) (k + j) \nless (k \nless a) \}. \end{array} \right.$
- 9) $(\text{num. } a > 3) = (0' a \bar{a} \cdot 0'; a 0' + 0) = \bar{a}; 0' (0'; a 0'); a.$
- 10) $\left\{ \begin{array}{l} (\text{num. } a = \text{num. } b) = \\ = (\bar{a}; a = \bar{b}; b) (\bar{a}; 0'; a = \bar{b}; 0'; b) \{ \bar{a}; 0' (0'; a 0'); a = \\ = \bar{b}; 0' (0'; b 0'); b \} \dots \dots \dots \\ (\bar{a} \ddagger a = \bar{b} \ddagger b) (\bar{a} \ddagger 1' \ddagger a = \bar{b} \ddagger 1' \ddagger b) [\bar{a} \ddagger \{1' + 1' \ddagger (a + 1')\} \ddagger a = \\ = \bar{b} \ddagger \{1' + 1' \ddagger (b + 1')\} \ddagger b] \dots \dots \dots \end{array} \right.$
- 11) $\left\{ \begin{array}{l} (a \text{ is gleichmächtig, of equal might with, } b, \text{ according to Herrn} \\ \text{G. Cantor's terminology: } a \text{ is "equivalent" } b) = \\ = (a \sim b) = \sum (z; \bar{z} + \bar{z}; z \nless 1') (b = z; a) (a = \bar{z}; b) \end{array} \right.$
- 12) $\left\{ \begin{array}{l} (a \text{ is } \infty) = (\text{the collection } a \text{ is finite}) = \\ = \Pi \{ (z; \bar{z} + \bar{z}; z \nless 1') (a \nless \bar{z}; a) \nless (a \nless z; a) \}. \end{array} \right.$
- 13) $\left\{ \begin{array}{l} (a \text{ is } \infty) = (\text{the set } a \text{ is "actually infinite," transfinite}) = \\ = \sum (z; \bar{z} + \bar{z}; z \nless 1') (z; a < a \nless \bar{z}; z; a). \end{array} \right.$
- 14) $(f \text{ is a function}) = (f; \bar{f} \nless 1' \nless \bar{f}; f).$
- 15) $(s \text{ is a substitution, permutation}) = (s; \bar{s} = 1' = \bar{s}; s).$
- 16) $\left\{ \begin{array}{l} (i \text{ is an } a \text{ of } j) = (i \nless a; j) = a_y = \bar{i}; a; j \\ \text{— } i, j \text{ representing individuals, compare 7).} \end{array} \right.$

- 17) { (The set a is put in *simple order* according to the principle $x) =$
 $= (x; x \Leftarrow x = 0' a \check{x} \check{x})$.
 18) { (The whole universe of thought is marshalled in a simple or-
 der, in a file or succession by $x) = (x; x \Leftarrow x = 0' x \check{x})$.

Dwelling for a moment upon the above, we shall here perceive represented and defined through the medium of the fund or capital¹ of denotation hitherto secured, quite a series of notions, fundamental for Arithmetic and Mathematics in general.

Before considering some of these in detail, a few remarks are required.

From 5) until 9) and at 16) I have, as concluding term, given the definition itself also in the shape of a binary relative. These then are "prominent (ausgezeichnete)" relatives, being only capable of one or other of the two "truth-values" (or "absolute moduli") 0 and 1.

The "*class*, assemblage, system, collection or *set*" having been defined by 5), I did not, from the middle line of 7) onwards, explicitly state that a , and may be b , ought to represent assemblages or classes, leaving this to be tacitly understood for fear of overloading the formulæ.

We next hit upon the definition of the lowest *natural* (i. e., positive integral) *numbers* 0, 1, and 2. Verbal Logic has hitherto proved incapable of defining even the *casus singularis*.

It is, of course, not practicable to enter upon the explanation and establishment of all these definitions one by one. I should like, however, as an example which can be easily understood, to point out the genesis of the definition of the number 2. The last line of 8) literally shapes into expression: There is an element i and again an element j ("another element"), differing from the former, such that (both are) each of them is contained within the set or assemblage a , whilst every element k differing from the one and the other i, j will not be contained within a . Evidently this is indispensable and sufficient whenever the set a shall consist of ex-

¹"Capital" is here to be taken in the sense of Adam Smith and Political Economy.

actly two elements. But according to the rules or Laws of the Algebra of Relatives, as developed in my book, the preceding prolonged intricate statement easily *condenses* into the forms given above it. Eventually there is but an expenditure of say six letters to be spent on defining "a pair" or the *casus dualis*. I wonder whether that can be styled waste!

In 11) besides the (relative) notions of "*equal might*" (between sets) you may observe as being pasigraphically defined the notion of "*Abbildung*" or one-to-one correspondence, the latter standing behind the Σ . That is to say: the sets a and b are to be called of equal might (multitude), whenever there exists a relative z which in that sense *images* (projects) the one set on the other.

12) gives the definition of *finiteness* (of a set). This, in accordance with Peirce, may be given independently by expressing the fact, that in passing from one to another through the elements of the set one must necessarily come back to an element already passed.

13) gives the definition of *infinity*, likewise independently in the usual manner: as the quality of the set to be capable of being imaged (projected) on a proper part of itself.

Both notions can be shown (by mere calculation) to be but negations of each other, their definitions being contraposed to one another.¹ Neither of the two definitions exhibiting or containing the least particle of negation, they furnish a good example for illustrating the falsity of the doctrine, still current among professional philosophers, of a distinction being logically possible between notions or marks (*Merkmale*, *notae*) positive and negative in themselves. I challenge any one of them, including Mrs. Franklin-Ladd (compare her review of my Vol. 1 in *Mind*), to decide which of the notions "finite" and "infinite" is the positive one and which the negative, whilst for such decision supplying reasons that appertain to the domain of Logic.

10) gives the explicit condition for equalness of number, i. e., for the fact that two sets a and b contain the same number of ele-

¹ See my papers in the *Nova acta Acad. Leop. Carol.*, Vol. 71, 1898.

ments, or that within each *equally many* individuals may be counted. This, of course, presupposes the finiteness of both sets.

The condition is set out as an infinite series of partial conditions, and in the shape of a *relation between both sets*. It may well be seen therefrom how well-founded is Herr DEDEKIND's remark: that the notion of the "number" of things is wrongly believed to be a simple one.

Into 17) the notion of the "simple order" has pasigraphically condensed itself from the marks that the Signori VAILATI and BURALI-FORTI have pointed out one by one, endeavoring to invest them with and dress them in the symbolism of the Italian school—a symbolism apparently not equal to such tasks and no match for our pasigraphic symbolism, supported as that is by so powerful a discipline as Peirce's Algebra of Relatives. It reminds one of stenographic briefness to notice that for a full investment and adequate expression the statement 18) an expenditure of only five letters is needed. Nevertheless, every person versed in relative Logic can read therefrom *all* the qualities of a simply ordered Whole, either whilst skilfully deducing them by conclusions to be drawn of necessity, or even at first sight, by mere inspection. Of course, supposing a flourish of any kind to be made, such merely shorthand logogram ("Schlüssel") would easily beat in briefness our pasigraphic expression, but then that which is most valuable in the latter, i. e., the fact (last mentioned) of its containing visibly condensed within itself all the marks of the notion to be represented by it, and therefore of its being capable to yield them again at any moment, would be forfeited.

With respect to the notion of "order" and its different "types" it would be well worth while to enter and enlarge upon the pasigraphic representation of the many notions with which Herr G. CANTOR has here enriched Science. We might, for instance, next show that the postulation: "there exists within the set α , ordered by the principle x , an element of lowest range" presents itself thus $\alpha \notin 0' \bar{x}; \alpha$, and that $\alpha(\bar{x} \neq 0)$ is the expression of this "initial" element, and so on. But the time at our disposal will not allow me to continue in that direction.

Similarly as in the foregoing we could now also pasigraphically define the statement

$$19) \quad (\text{num. } a = \text{num. } b + \dot{1}).$$

One would thus for the realm of numbers succeed in constituting a certain Relative :

20) $g, = \text{by } \dot{1} \text{ greater than-}$
by means of which, though not very simply, is to be represented also the Relative :

21) $t = \text{Teiler von-}, \text{ divisor of-},$
or else, if it be preferred, this one: $\tilde{t} = \text{multiple of-}, \text{ Vielfaches von-}.$

Then we shall have :

$$22) \quad \dot{1} = t \dot{+} 0, \quad \dot{0} = \tilde{t} \dot{+} 0,$$

and again, for example :

$$23) \quad \left\{ \begin{array}{l} r = (\text{relatively prime with-}, \text{ teilerfremd mit-}) = \tilde{t} \dot{+} (\dot{1} + \tilde{t}), \\ (m \text{ is prime with } n) = (m \leq r; n) = \tilde{m} \{ \tilde{t} \dot{+} (\dot{1} + \tilde{t}) \}; n, \\ \text{Prime number} = (1' + \tilde{t}) \dot{+} \dot{1} = (t + r) \dot{+} 0, \\ (\text{Greatest common divisor of } m \text{ and } n) = \\ \quad = t; m, t; n, \{ \tilde{t} \dot{+} \tilde{t}; (m+n) \}, \\ (\text{Least common multiple of } m \text{ and } n) = \text{idem, } \tilde{t} \text{ in lieu of } t. \end{array} \right.$$

And thus to be continued at pleasure. With these and suchlike forms it is possible to *calculate*, and inferences regarding the notions they represent, may be drawn and extracted from them. This latter could not be effected with mere shorthand logograms, such as is for instance Peano's $D(m, n)$ for the notion next to the last in 23).

The notion of absolute prime number being doubly represented above (for the realm of the whole numbers), the first representation states: prime number is a number which stands to each number, except the $\dot{1}$, in the relation of either being identical with it or being no multiple of it. The second states: prime number is what to every number (without exception) stands in the relation of either being a divisor of it or being relatively prime with it. And, on the strength of the pasigraphic structure (not here given) of the Relative t itself, either one of these two representations will be capable, moreover, of being transformed into the other.

For the sake of throwing a momentary glance on topics other than Arithmetic in this place, supposing the universe of discourse 1 to mean Space, the definition of a geometrical point may be given :

$$24) \left\{ \begin{array}{l} (x \text{ is a point}) = (x \vdash 0) \prod \{ (x \Leftarrow u) + (x \Leftarrow \bar{u}) \} \\ \text{or, in another form (after Peirce)} = (x \vdash 0) \prod \{ (u \subset x) \Leftarrow (u = 0) \} \end{array} \right.$$

In its first shape our definition settles the "point" to be such a part of space, differing from nothing, which to any part u of space stands in the relation either of being wholly contained within it, or being wholly without it, that is to say, being wholly contained within the remainder of space \bar{u} . I leave to the reader the interpretation of the second form of definition, which has been already reduced by me to the former in my Vol. 2.

Finally a word anent the Pasigraphy of human *relationships* embracing as well those of consanguinity as those of affinity and forming no unimportant chapter in the *corpus juris* for the student of law. In addition to a few of the signs of General Logic above set out, there are only requisite *two* specific symbols of Relatives, for representing discriminately and exhaustively all these relationships in the concisest possible shape. These two are :

m = male (an absolute term),

and

c = child of- (a relative one).

Mankind consisting of two sexes then \bar{m} = not-male will denote female, and \bar{c} , as before mentioned, will equal "parent of-." The universe of discourse 1 = $m + \bar{m}$ then consists of the Persons of human society in the Past, Present, and Future. However, for rendering fully accessible to our pasigraphic system also the relations of affinity (i. e., those by marriage only), to every childless married couple must be ascribed one "potential child." True, that for completely realising the ideal of Pasigraphy it might be demanded that, again, the notions "male" and "child of-" themselves should be reduced to primitive notions of a simpler breed. But such a thing might only be hoped for when Zoölogy and Physiology should have developed to a much higher degree of perfection. Meanwhile something yet is to be won if we plainly admit these two notions m and c as primitive notions and henceforth use them as building stones.

Then the following will be the pasigraphic representation of sundry relations:

- (Maybe only half-) brother or sister = $0'.c;\checkmark$,
 Full brother or sister (Geschwister) = $0'.c;m\checkmark.c;\bar{m}\checkmark$,
 Fullbrother = $0'.m.c;m\checkmark.c;\bar{m}\checkmark$, Full sister = $0'.\bar{m}.c;m\checkmark.c;\bar{m}\checkmark$,
 25) Stepchild = $\checkmark.c;\checkmark;c$, Father = $m\checkmark$, Mother = $\bar{m}\checkmark$,
 Consort = $0'.\checkmark;c$, Husband = $0'.m\checkmark;c$, Wife = $0'.\bar{m}\checkmark;c$,
 Nephew or niece = $c;0'(c;\checkmark)$, Mother-in-law = $\bar{m}\checkmark;0'(\checkmark;c)$.

All these multifold connexions have been most profoundly studied by Mr. Alexander MACFARLANE who has, for instance, answered the question: which relationships (being of the second degree) are excluded (prohibited from existing) by the English Law that forbids a man to marry his deceased wife's sister. With such-like expressions as those already given any kind of problems may also be solved mechanically, by mere calculation, as, for example, this: a lady, questioned about a photograph in her album, replies: "you know that I have no daughters. Well, this person's daughter's son is the father of one of my grandchildren." How was the original of the portrait related to the lady?

Macfarlane, however, because of his repudiating Peirce's Algebra of Relatives, or at least abstaining from its use, did not clear a certain reef. Whereas in the expressions by him established, that yet are somewhat different from the above, he did not succeed in excluding their "reduced meaning"—as he chose to call it.

The gist of the situation may already be clearly perceived in the well-known riddle for children: My father has a son who still isn't my brother; who is it?

The "reduced meaning" of child of the parents of somebody is this somebody him- (or her-) "*self*" (1'), and therefore the supplementary appending of the sign $0'$ = "another than-" to $c;\checkmark$ is indispensable for correctly forming the notion of brother or sister.

Turning from these special investigations of English origin and leaving untouched several isolated attempts (as for instance that of Herr FREGE, who heedless of anything accomplished in the same direction by others, took immense pains to perform what had already been much better done and was therefore superseded from

the outset, thus delivering a still-born child) save the fundamental work of Mr. PEIRCE in the United States and his German and English precursors among whom BOOLE and DE MORGAN deserve first mention, the aims of Pasigraphy hitherto have found assiduous promoters only in *Italy*.

The meritorious periodical *Rivista di Matematica*, edited for five years by the eminent mathematician Signor PEANO, our chairman on this occasion, together with the supplementing *Formulario*, are mainly devoted to its purposes. And in this and other periodicals through a group of keen Italian investigators quite a series of branches of Analysis and Geometry has been worked over with pasigraphic intention and enormous application. All that can be done with the Boole-McColl "calculus of equivalent statements," and that is certainly much, appears almost wholly to have been thereby accomplished—though, regrettably, in a greatly diverging system of denotation. On the other hand, in its general features the present phase of the Italian pasigraphic movement is characterised by the non-use hitherto of Peirce's Algebra of Relatives. Against turning the latter to profitable account the denotation system adopted by the Italian school indeed seems almost to form an obstacle. Their capital of denotation lacks the most general primitive notions, which in the Algebra of Relatives already exist and are tolerably well investigated with respect to the rules of their combination. For these missing categories numerous invented and arbitrary logograms (occupying 5 printed pages of Peano's *Table des signes* and still on the increase) prove insufficient substitutes and are but poor makeshifts. To conclude here, in short, I may venture to apply to them the parable, put forth by Professor MINKOWSKI in his address when introducing the proceedings of our Section, concerning those who persist in still using sailing ships whilst steamboats have already been invented, constructed and are waiting at their service.

If I have successfully shown how with the same means the notion of infinity and of the greatest common divisor, equally well as that of mother-in-law, can be expressed, then surely it will be admitted that Pasigraphy has now indeed emerged from the status

nascendi and that its ideal must have been realised at least to some extent.

In the cases—ever rare—when humanity has succeeded in essentially realising an ideal, as a rule its subsequent aspect will widely differ from the form in which it hovered before those who conceived it first. So in this case. Already we can say thus much, that Leibniz's prediction: "*scriptura haec universalis aequae erit facilis quam communis*" is scarcely likely ever to be fulfilled, and that Descartes's hope, that by its aid a peasant would then gain a deeper insight into things than is now possessed by a philosopher, will probably never be realised.

It is in the *calculus ratiocinator* that the difficulty lies! The higher parts of Logic present such an abundance of problems ranging among those of the very highest degree of intricacy, and mastering the Algebra of Relatives—accessible only to serious workers—is so little easy to attain that it may well never become common property, always remaining the privilege of but a few favored thinkers.

In conclusion, and returning once more to the 5 primitive notions 1), I have permitted myself, in selecting them, to be led by regards of convenience for the purposes of my lecture. I have been very far from implying, however, that their number may not possibly be further reduced. As a matter of fact our "category" of conversion seems—by means of the definition

$$(i \in a; j') = (j \in \bar{a}; i)$$

wherein i and j in the sense of 7) represent individuals—itsself to be reducible to the *four* remaining primitive notions, provided only that the whole set of the "Definitions" be systematically arranged in a proper manner.

Then the "four elements"

same, and, not, of

or identitas, intersectio, negatio, relatio, in intimate association will both form life and sustain the world intellectual.

ERNST SCHRÖDER.

KARLSRUHE IN BADEN.

THE SOCIAL PROBLEM.¹

(CONCLUDED.)

WE HAVE now to recapitulate the principal propositions that flow from the developments which we have given in the preceding part of this discussion.²

(1) Nature is an unconscious mechanism ; she is indifferent to the phenomena which take place within her domain and to the creatures which live there ; and man is neither more nor less than other animals. But man happens to possess an advantage over animals, and from this superiority he derives all the benefit that he can. During the time that he lives, his concern for his conservation and happiness is his whole care. He has an ego which protects him against his own weaknesses, over which his reason and sentiment have not always full control. A very large number of his acts are unconscious. Society is a means of existence devised and exploited by man,—a means upon which he counts for increasing his power, for diminishing his sufferings, and for obtaining the greatest possible amount of satisfaction. Insensibly society has been transformed into a corporation which embraces both the present and the future.

(2) Societies have sprung from two sources : from the family, the members of which staid together ; and from indifferent assemblages, which were at first altruistic and were afterwards based on interest. External defence was the first stage ; internal defence the second. The progression was accomplished by force of circum-

¹ Translated from Dr. Topinard's MS. by T. J. McCormack.

² See the July *Monist*, pages 356-395.

stances without agreements of any kind. Contracts did not arise until later and then partly in consequence of some combat or crisis. Such was the case of the English barons and their followers forcing upon King John the Magna Charta of 1215, and such, too, was the case of the Pilgrims of Plymouth Rock in forming a constitution when they took possession of the land granted to them by James I. The "contract" of Spinoza and Rousseau is a mere theory, but nearer to the truth than the "living organism" of the positivists. In every democratic society there is a virtual pact of some kind between the two contracting parties: society has its duties to fulfil towards individuals, just as individuals have duties to fulfil towards society, or towards their fellow-beings—two equivalent terms, for society is the aggregate of one's fellow-beings. But to-day the contract tends to become more formal; the drift, now, is to submit not only the constitution but even the chief laws of the legislature to the sanction of a referendum.

(3) Societies have evolved empirically according to circumstances and individuals, or, to use a phrase of the day, according to the principle of *laissez aller*. Individuals, by nature very different, have played their part conformably to their special organisations, the strongest causing all things to centre about their personal interests, a small number only zealous for the interests of all. The results in different directions have thus only feebly responded to the end for which society was established, and if among these results selection, which operates with things as well as with animals, has made for the profit of the most prosperous societies, the reason for it is that nature never loses control of her rights, and always gives her sanction and justification to the most powerful.

Among these results there are many that are good. These are, for example: that permanent form of association, lasting from generation to generation, analogous to a stock-company, which compels the society to shape its action with regard to the future as well as the present of the species; the hereditary capital which is its result, which is exploited for the greatest welfare of all, and the profits of which must therefore be distributed with equity; the softening of manners; the pleasures of the intellect, reckoned

among the most desirable of superfluous needs ; in fact everything which may be recapitulated in the phrase " the progress of civilisation."

But there are evil results, too. For example, that horrible militarism to which external defence still forces us, and which, when war breaks out destroys at a blow all notions of morality. Then that internal scourge which comes from the results of the struggles of ancestors being perpetuated among their descendants, and from our being responsible not for our own conduct, but for that of our forefathers. Next the division of society into strata, the higher enjoying from birth a position and wealth which exempt them from all effort, the lower frequently conquered before they have fought, and predestined to misery and suffering. I have already spoken at sufficient length of the condition of the lower classes. I will but add a word. Whilst among the favored classes the family is the sanctuary and the focus of all joys, among the proletarians of Europe it has been almost totally obliterated. The father and mother labor, each in his sphere ; the latter is unable to give to her child that initial education which is so decisive for the whole of life ; frequently she is obliged to place her child in a foundling asylum ; the boys and girls of more advanced age are scattered in the workshops, or roam the streets exposed to all sorts of bad examples and temptations ; even at night they scarcely come together and make the acquaintance of their domestic hearth.

(4) The method of life in common was adopted by man with a view to increasing his means of action, and affording to his faculties the fullest capacities of development in the direction which claims his nearest interests. He seeks in this way to free himself from the performance of certain general services which naturally fall to the lot of all, and which would distract him from his immediate occupations. He desires to work out his own happiness in his own way, to be responsible for his acts, and also to enjoy the fruits of his responsibility. Society, therefore, is bound to allow him a maximum of liberty in order that he may have a maximum return ; its reason for being would be annulled if the individual were lessened by the social state.

(5) Now it is by competition or struggle alone that the individual achieves his fullest value and finds the employment in which his faculties are best utilised. It is in struggle that the higher individual variations of which we have spoken find their fullest expansion, and that the mean variations, and possibly also some of the lowest, are either heightened or are put to use under conditions which are suitable to them. Society cannot think for a moment of eradicating struggle. From its own point of view as desiring the welfare of all, or as a commercial and industrial association working a capital and obligated to declare dividends, it ought even to encourage it. The over-production of all things necessary to material life, to welfare, and to intellectual enjoyments, which are the result of the labor of individuals, redounds to the profit of the whole social mass. The activity engendered by the struggle, selection wanting, is the agent which perfects the species. Hitherto that activity was restricted to certain classes, the lower had few needs and produced nothing beyond what was actually necessary. Now this activity is extended; every one wishes to have his share of the superfluous satisfactions. We have seen that it is not absolutely certain that intelligence has increased since antiquity; the reason for this probably is that this activity was partial and poorly directed. To-day, when it is becoming general and is growing in quantity, it is impossible not to encourage it. Struggle, of which the results are no longer sanguinary, but lead to more or less welfare and satisfaction according to the activity employed, is the highest necessity both for the individual and society.

6. There are philanthropists who would replace struggle with peace and universal fraternity. Instead of the formula of physiological justice,—viz., "to each one according to his faculties, his deserts, and his works," three synonymous terms as here used,—they say, with their metaphysical conception of absolute justice, "to every one according to his needs," that is to say: to the indolent, lazy individual who shirks work, as much as to the active, useful individual who produces more than he needs, who, it is true, enriches himself but at the same time enriches the mass of his fellow-beings. This would be absolute hypothetical justice,

clashing with the only demonstrated justice, the only one having a claim to the title of natural justice. For these philanthropists the ideal of society is that of a great family of which the members are closely joined and solidary, as in completely unified animal colonies, enjoy all their liberties without restraint, share all things with all, live each according to his own tastes, and satisfy all alike their immediate and superfluous desires, all the peers of the others. But this programme is self-contradictory in certain of its parts, and it is contradictory to nature, the individual, and the social idea itself. It is a Utopia impossible to realise even in the most distant future, and as a whole not even desirable. It is equality pure and simple. Establish it, and in a month, in a day, in an hour, it will no longer exist. In a word, what here around me in the schools of jurisprudence is called the moral law is contradictory to the law of nature and but a fiction, a generous conception of our cerebral sensibility.

7. We have not yet stated our conclusions regarding the questions which were mentioned at the beginning of this chapter, but they are suggested in so forcible a manner that the reader has himself doubtless formulated them a score of times. The realities of nature and the necessities of life in common are irreconcilable if we refuse to see things as they really are and if we seek to model practice on absolute conceptions which have no basis but desire, sentiment, and imagination. The individual, unmodified by habits or impulses more or less unconscious, but left to his own nature and controlled only by his animal ego, recognises only his own interest and craves for liberty pure and undiminished. Society is a complexus of concessions to the common cause. The individual grants these concessions to society and keeps his contracts, but only so long as he finds profit in it, or because he is not the strongest. Whenever he reasons coolly, where there is no outlook for punishment, where there is no fear of opinion, nor of the mediate or remote effect of his conduct, his animal and egoistic nature appears. Society can only master him by force, and discipline him like a soldier in an army.

But the realities of nature and the exigencies of society admit of easy reconciliation if instead of dashing ourselves to pieces on

the two rocks of animality and the absolute, we steer between them; when we submit to accepting what we cannot avoid, for example, the organic and intellectual inequality of men and the absence of real justice; and if we do not lose from sight the definition of Montesquieu: "Laws are the necessary relations that are derived from the nature of things." This means that rules and laws, being the sanction of the best possible relations between individuals and society, should not be left to the mercy of empiricism, the caprice of a monarch, of a multitude, or of any form of universal suffrage; that they should be dictated by reason after light has been received from all possible sources; that between all the solutions which present themselves there is one which is best adapted to existing conditions and is the necessary relation sought. Jean Jacques Rousseau has defined law to be "the expression of the general will." This is not more exact than if we said, "of the will of a monarch or of a parliament"; for will may be poorly illuminated, blind, unintelligent, passionate, and in disaccord with utility as rightly understood. The "necessary relation" of Montesquieu can be determined only by a perfect knowledge of the subject by men who are carefully prepared, independent, and animated by a holy love for humanity, by men who will apply their best intelligence to seeking the solution of each problem duly studied, who will weigh the *pros* and *cons*, the advantages and drawbacks in each case,—by men versed in social science and its different branches, notably the science of law.

Evolution left to itself has yielded, as we see, both good and bad results. The latter must be amended, even if we must go to the quick. It is incumbent on man to take matters in hand himself and to direct their course. He knows the difficulties to be overcome, he knows what he has to renounce and what he has hope of obtaining. The human species in its duel with other species and with nature has won many victories. Man has but to continue his conquests and to introduce into his efforts method and logical consequence; he has found a way of appropriating certain of the forces of nature, of adapting numerous vegetable and animal species to his wants. It is impossible to suppose that he is not capable of

organising a society as he judges best and, if necessary, of transforming sufficiently his own nature.

Let us pass to the applications, in broad outlines. We will suppose a society at the stage at which our present civilisations are, of the average size, and democratic; we shall not consider others. We leave aside the United States, which was founded and developed under exceptionally favorable conditions, which did not possess the fixed routine of Europe, which adopted, at a single stroke, communal autonomy and the autonomy of states, and which is only faulty in point of federation, in embracing too many different regions, and too many dissimilar interests. What would be the functions of such a society, and what would be its attitude towards those for whose greatest happiness it was created?

The first thing which it must bear in mind is that the total mass, the general interest, alone exists for it; that the parts of this mass, the particular interests, figure only through the part which they take in the general functioning of society, and that individuals are molecules only in the pseudo-organism which it is called upon to direct. This is the principle of the unity of state, and the only way to comprehend the "Reason of State" and the Secret Funds which are admitted in very exceptional cases, in the present state of things, for the public safety. The members of the parliaments, whatever be the manner, felicitous or unfelicitous, in which they are appointed, represent the country in its entirety and not any particular circumscribed part of it. Their lot is to pass general laws which apply to the needs of the mass without stopping to consider exceptional individual cases. When their duty is accomplished, which is to grant equality to all before the law, and, more exactly, equality of advantages and disadvantages resulting from necessary laws,—they can only submit to the inevitable injustices which they here and there produce. Thousands of innocent human beings are sacrificed in case of war, and in the interior of the state, too, there are untold necessary victims of the universality of laws. The legislator has an enormous responsibility. What he decides should be accounted infallible, although he may be in error.

He must act for the best, knowing that he cannot attain perfection, however much he may be inspired with ideal conceptions. But what he should also never lose sight of is that each of the persons under his administration has in himself the sentiment of relative justice of which we have spoken, of "that which is his due," and that this justice implies the natural right to insurrection inscribed in the declaration of the rights of man a century ago.

The functions of the state are divided into essential and facultative, the first falling under three heads: (1) external defence; (2) internal defence; (3) general services.

External defence. This is of two kinds: military and economical. The former gave rise to the first societies, which for a long time remained at this stage. Unfortunately its counterpart followed—attack and then conquest. Militarism resulted, becoming a need, a passion for domination, for rapine and glory, growing worse with time and falsifying the entire mechanism of society. Even to-day it is the greatest obstacle to the serious progress of humanity. So long as the ethics practised in time of war is so violently opposed to the ethics professed in time of peace, it will be impossible to inculcate in the minds of individuals that there is but one ethics. And yet militarism is a necessary evil which we cannot avoid, a devouring cancer which we cannot cure. The first need of a nation is to defend itself and to make itself respected, in order to live. War absorbs the best wealth of a country, it decimates it, it leaves behind it nothing but ruin, it makes of man a ferocious beast. Yet despite it all, we must be ready for it. On the fatal day all the members of a society are here solidary; all devote themselves as a mass to the common safety. The state, even in time of peace, has an army to support; vessels, cannons, munition, ports, fortifications, strategic roads, hospitals, special schools, an entire administration to create, watch over, and recompense. This function alone, bearing as it does upon a large number of points, requires a complete centralisation and alone absorbs a great part of the action of the state, gives to it an excessive influence and enables it to mingle in the life of individual interests more than the principle allows. By its budget it weighs down heavily upon the nation; by its ob-

ligatory service in countries which from their geography have no natural defence, it turns from life at the decisive moment of existence the whole able-bodied masculine population. Militarism is the worst of scourges but a necessity of the times, to which we must submit.

But there is not only the war with cannons. There is another species of warfare, which has been termed peaceful, and which is conducted by its side. The extension of exchange, the facility of communications, has in modern times swollen it to such proportions that the state has been obliged to interfere and to protect its members. Economical, commercial, and industrial competition between individuals has overflowed the frontiers of nations and become international. If we consider the general interest of humanity alone the system of protection against other countries is wrong. Free exchange, the free circulation of the means of existence, drawn as water in communicating vessels to the places where they are scarcest, is the true law. When a country does not produce these means of existence, or does not produce them in the desired form as regards cheapness and quantity, it is reasonable not only that it should accept them from its neighbors, but also that it should demand them, and that in return it should furnish to them what it produces cheaply and abundantly. Protection is a device for forcing a country to be self-supporting and for creating industries of its own, for it is undesirous of being tributary to foreign markets. The reasoning is correct from a national point of view, but it proves that the sacrifices for the general welfare which society exacts of individuals in its own sphere are refused for the common welfare of humanity. It is always the question of the two schemes of ethics—one for ourselves and one for others. But there are products which one does not possess at all and for which we must have recourse to others. The United States are a new country, rich in mines of all kinds, capable of producing everything of which its people have need. They may permit themselves the luxury of dispensing with the rest of the world, for they are in the highest sense a social joint stock company. But in Europe the situation is different. The various states are obliged to supplement one another. England in its

insular condition has long since learned that it cannot with its agriculture contend on an equal footing with the remaining world and that it is obliged perforce to become industrial, trading, and distributive, as were formerly the Phœnicians, the Genoese, and the Dutch. It is in its proper sphere. But is it not incumbent also on the other peoples of Europe to band together, and upon this basis to take the first step towards the United States of Europe? At present societies protect themselves by the aid of export and import bounties, subsidisation of merchant marines, instructions to consuls, and especially by the aid of treaties which the Powers wrangle over exactly as individuals do.

But if rival societies have hostile interests, fortunately they have also common interests, and here there is ground for understanding, which is destined, we are convinced, to assume greater and greater proportions. From this arises a host of treaties of all sorts regarding postal communications, weights and measures, money, literary property, the extradition of criminals, the establishment of sanitary regulations, and so forth. The sphere of jurisdiction of the state is, therefore, even thus far and for exterior affairs already considerable. In France, if we deduct the interest of the national debt, the budget of the exterior is alone one-half of the total budget.

Defence of the interior.—This is the second function of the state—the defence of individuals against one another, against the causes of interior calamity, and against themselves.

The first outweighs the others. It is the protection of individuals who restrict themselves absolutely to the exercise of their recognised rights and observe the laws, against those who violate these rights, trespass upon those of others, and break the laws. It embraces assault, material obstruction of one's actions, slander, etc., infringement of property rights and of the right to labor, the violation of contracts duly attested, etc. One of the sacrifices imposed upon the individual being to refrain from administering justice himself, save in cases of self-defence, society is obliged to discharge for him this function in some manner.

The second class of measures for interior defence is concerned

with salubrity and embraces measures for the prevention of diseases of men, useful animals, and plants. The third class is concerned with the protection, in exceptional cases, of the individual against himself. Evidently the individual is master of all of his acts which concern only himself; he may even commit suicide. But when he is obliged to apply to professions whose practice requires special knowledge and ability, of which he is not capable of being a judge, and which may have the gravest consequences, surely the state should come to his help and protect him against his own ignorance. Such professions are those of medicine and pharmacy, of law, of navigation, and even of civil engineering and architecture. The practice of these professions must be sanctioned by certificates or diplomas, awarded, or at least stamped, by the state. Probably the day will come when the public will not be deluded by sensational advertising and charlatanism, but that day is still far distant.

The economical protection which we saw at work abroad has its complement in the interior of the state; the one brings the other in its train,—both are to be regretted. Bounties are granted here and there for supporting national competition,—in France, for example, upon sugars and silks.

By the side of these is seen another species of economical protection which is absolutely condemnable, and which cannot be explained except by the personal bias and interests of legislators which ought never to exist. I refer to the special protection of some one industry, some one region, some one group, or even some one class. It rises from the arbitrary and unequal imposition of taxes, made either through partiality or ignorance.

This brings us to the reverse aspect of the protection of individuals, to the total abstention of the state from everything which constitutes a private act, from everything which bears upon the normal course of life, and to that fierce struggle which must leave individuals to their own risks and perils.

In the face of that struggle, which we have shown to be at once legitimate and necessary from the triple point of view of progress broadly viewed, of society considered as the administrator of the common capital and the distributor of its dividends, and of the

individual seeking to exercise all his faculties and to bear the responsibility of all his acts; in the face of this struggle, in which the result is no longer selection by death but the need of enjoyment and the desire for a better position in life,—a struggle of which the effect is to disengage the higher individual variations for the general profit of the whole social mass and to furnish that employment which accords best with the average and lower variations,—in the face of this struggle, I say, the attitude of the social body is distinctly marked: absolute neutrality, the awarding to every one of a full recompense for his efforts and the leaving to him of all the consequences of his failures, however they may have come about. Relative individual justice requires this; the intermeddling of the state in the struggle would be injustice. Men are unequal by the fault of nature; society has simply to bow to the fact; all that it can do is to seek to render the combat loyal and courteous, and if possible to prevent the conqueror from absolutely crushing and destroying the conquered. Without detriment to the principle of non-intervention, it may also prohibit the struggle on the part of those who are plainly without arms, and to prepare for it those who are not so. Let me explain myself.

Society should have asylums for idiots and the insane, for congenital cripples and non-developed children. It should gather under its paternal care foundlings and orphans, assume charge of and prepare for life during the necessary period of time the children of fathers and mothers who are incapable of fulfilling this task. What it should do or seek to do is, above all, to equalise as much as possible the external conditions of the combat at the start. It is customary in a duel for the adversaries to have the same arms, the same kind of ground, the same clothing as nearly as possible, the same kind of shoes, etc. The rest is left to the valor and skill of the combatants. It should be the same in the social struggle. Birth places the combatants in very different positions: the one has capital, property, education, rank; the other has none; the one has all the chances of conquering; the other all the chances of being conquered. In a word, the sons are not exclusively responsible for their own acts; they are responsible for their fathers' and

ancestors', and for the situation in which the latter have left them. This is a monstrosity,—that which from the beginning of society has weighed down the most on evolution, as we know. But, it will be said, this is attacking inheritance, consequently the family, the right of every one to labor for his children, which is one of the most powerful main-springs of human activity. Unquestionably and precisely it is an instance of the impossibility of reconciling everything. Whatever may be the solution, justice is wrecked on the one side or on the other. There is no amelioration possible except by adopting a middle course: suppress all inheritance *ab intesta* outside of direct ascendants and descendants and of the wife and husband, that is, outside of the immediate family, and restrict in the same sense the right of testamentary disposition. Bequeathable property would revert to the state and enable the state to abolish all taxes which now press so heavily on the labor of men in society.

General services.—The department of general services is the third essential function of the state. Everything which requires the co-operation of all, upon which it would be difficult to come to an understanding, or which would divert the individual from his personal occupations implies a central direction and is the province of the state. In truth, all the functions of the state fall under this last category, excepting war, where every one may be put in urgent requisition. Such are the preparation for war itself, the exterior economical defence, the interior defence with its three principal forms, with its two organisations of police and justice, education and public aid, of which we will soon speak.

The general services to which we refer at present are highways, canals, railways, not connected with war but with the internal prosperity, with the transportation of the means of subsistence and of travellers, the postal and telegraph service, depots and markets, forests and parks reserved for general recreation, although collaterally exploited for the needs of the state, and finally the finances, which we meet with everywhere and which are the contribution of each to the common expenses, being essentially (1) a fixed part, equal for all, the non-payment of which brings on the loss of the

advantages connected with the rank of citizen ; (2) a supplementary part proportional to the successes won in the struggle and to the enjoyments obtained,—that is, to one's fortune.

These services are of two kinds : the first are permanent in character and require a corps of employees of different grades, which constitutes properly the administration ; the second are intermittent and are evoked by the occasions of the moment ;—they may be let out by private contract, at auction, by governmental concessions, and by franchises, etc. The latter have the greatest possible extension. The rule is that the state should never compete with private enterprise, and that it should always have recourse to it unless there is some serious objection. The state, however, is responsible ; in principle it performs the work, it directs its course, supervises its execution, even when it avoids direct participation. It has been proved, furthermore, that work undertaken directly by the state is more onerous, requires a longer time for its completion, and is generally less thoroughly performed ; the responsibility of the state is too widely divided, or rather it is only nominal : its employees have not a personal interest in doing their work better—they take no serious risk. The work of man receives its value from the prospective remuneration, proportionate to the care which he bestows upon it, and to the perils which he fears. The proletarian who works by the day or the year does not labor as the individual does who is responsible to himself, who follows his own ideas, who knows that he has chances of losing as well as of gaining and that the good as well as the bad outcome of his labors depends upon his personal attention and activity.

The three functions of the state which we have just recapitulated, concerning especially actual individuals, are strictly speaking the only ones which are obligatory. But the state, being a permanent body having a paternal supervision over the welfare of its members, and being under obligation to look out for the morrow, an irresistible drift has extended its field of action for the better or for the worse. The material which we have to examine falls under two headings.

COMPASSION is the first. It is the sentiment of pity which so-

ciety is supposed to feel for those who suffer through its fault, or through the fault of nature. Society, strictly speaking, is not justified in this feeling: first, because not having a right to interfere in the consequences of the normal struggle between individuals or to modify personal responsibility, it is bound to abstain; secondly, because to interfere with those consequences and with responsibility is to attack the stimulus to all activity and all progress, and so to run counter to all that goes to the making of wealth; thirdly, because, if the individual has a nervous system and an apparatus of sensibility which moves him to make a matter of sentiment out of his risks and perils, to represent to himself the sufferings of others and to act as if he felt them himself, society possesses no such organisation. Society is comparable to an employee charged with a certain labor to perform, or to a manager of a business who has to think only of the dividends to be distributed,—it reasons only with figures and cannot yield to the stirrings of the heart.

Nevertheless, the fact cannot be disguised that if a community is comparable to a stock company administering a capital in the name of its stockholders, it can also be compared to a society for protection and assurance against the risks of nature. If the strong seek to lord it over the weak, the latter demand protection; the strong and young of to-day may be the weak of to-morrow and the old man of the day after to-morrow. When fathers expect children, are they certain that the latter will be favored by nature? Does not disease attack all? Therefore it is to the general interest to insure against the unknown, and nature being wanting, for society to assume the functions of providence. Furthermore, the sentiment of compassion is so imbedded in the heart of man that no voice is raised in opposition when assistance is made one of the accessory functions of society. The only difficulty is the exact measure to be meted out, a measure which it is difficult to fix as a general rule. The first consideration is not to give to the vanquished the joys to which they have not a right, and not to strip the vanquishers of the entire satisfaction of victory. Permanent or passing aid must not be converted into an encouragement to idleness or a premium upon vagabondage. We said just above that society should witness im-

passively the struggle between individuals, as did the heralds of the Middle Ages ; that it should see to it that every one on his entrance into the arena has fair and equal outward chances, but that it must be able, like the Cæsars at Rome, to stop the final and useless massacre of the vanquished. The doctrine of compassion would authorise society to do more ; it would suffer her to nurse the wounded, to assuage the suffering of adversaries put *hors de combat*. It follows that the department of public aid so-called, that is to say, of hospitals for the sick and homes for adults,—for we are not speaking here of children, idiots, insane persons, and cripples,—for civil and military invalids, for widows and paupers in given cases, are legitimate. And yet, as we said in a previous chapter, many among those assisted will remark : “Why then save and work for more than our immediate needs if our morrow is assured?” I do not speak of vagabonds or tramps or of mendicants by profession ; with these it is necessary to deal severely. “The benevolent action of charity can only be compared to the harm that it does,” says M. Émile Chevallier.¹ Aid is not a personal right for any individual,—this must be impressed upon the mind,—but a disgrace for the person who is the object of it. Every hand extended, every succor received, saving certain well-established exceptions, must be considered as a disgrace, must implicate the loss of civil rights lasting until rehabilitation. To reconcile all this, we shall recapitulate as follows : it is true, succor must be extended to the unfortunate, the old, the infirm, the vanquished in the struggle for existence, those whom circumstances and their natural inferiority rather than their conduct have ruined, those whom wounds have prematurely rendered unfit for the arena ; but the succor should be given with discernment day by day and be reduced to a minimum ; it should be given after inquiry, in just the necessary amount and no more. Since compassion, which nature does not possess, and individual justice, which requires that each should bear the conse-

¹ Émile Chevallier. *La loi sur les pauvres et la société anglaise. Couronné par l'Institut.* Paris, 1895.

quences, bad or good, of his acts, are contradictory, therefore compassion should not be made a clog upon justice.

There are two systems of charity : one administered by the state ; and one by private persons or associations. The two may be administered simultaneously : the first in incontestable cases—cases of the infirm and the insane without support ; the second in cases which are more doubtful—as the case of those who have fallen in the struggle, etc. But there is a remark to be made with respect to state charity. Every time a supplementary function is added to the work of the state, the money always comes from the pockets of the tax-payers, and it is in reality they who perform the service. The question comes back therefore to this : will the state distribute its aid better than private persons or corporations? In the first case it is naturally the function of the county, township, or parish, and not of the central authority.

PROGRESS is the second supplementary function of the state. In this point of view and as the heir of a physical, intellectual, and moral patrimony, from which all its members draw and which it must transmit, augmented and bettered, to posterity, society has several questions to consider. Should it, or should it not, look with favor upon the increase of its population? Should it stimulate individuals to advance in the path which sociologists declare the best for multiplying its power of production and for most justly distributing the fruits which flow therefrom? Should it endeavor to modify its customs in the most favorable direction, in the direction which gives the most satisfaction under the conditions of life in common? Should it seek to impress a definite direction upon the best habits of society, upon character, upon manners of feeling, thinking, and acting? And, in such a case, what shall be the means employed? Shall they be employed directly or indirectly, and upon what shall they be based?

The answer to the first question is not ambiguous in the present state of Europe ; men are necessary for defence. But suppose war should be abolished : then an excessive population would be a drawback ; men, all other things being equal, will, in a given space of territory, be happier when their number is small than when it is.

large. With regard to the other questions there is much to say. I shall take but a few examples.

The right of assembling together, the right of association which flows from it, are among the rights which the French Revolution regarded as inalienable. They have given birth to society itself. It would be strange if men could not band together now as they did for the first time and under the same influences—common interest and sometimes sympathy. In our day the principle of association has been considerably extended and is the force from which the future has to expect the greatest beneficence. There are commercial associations of a small number of responsible members or of an unlimited number of mere stockholders with responsibility limited to their holdings; industrial associations for protection, circulation, or consumption; political, scientific, and religious associations; professional syndicates of employers or workingmen, associations for education, charity, sport; and hundreds of others having the most varied objects. Some are mere instruments in the struggle for existence, employed by individuals, with which the state has nothing to do but which it generally must know of, so as to assure itself that their doings are not in violation of its laws. The others have for their object various public utilities, for which the state, if these associations were lacking, would have to care,—associations which consequently a state has the best reason to encourage. Every liberty, in fine, should be granted to associations, which are a form of progress, provided they infringe in no way upon the recognised liberty of individuals. In the eye of the state they are simply collective individuals having the same rights and the same duties as single individuals.

A serious question, however, presents itself. The individual is the present social difficulty, the enemy to be adapted to the necessary customs, the element of revolt which is always disposed to substitute its own personality for that of the state. We have seen that society, in consequence of its obligation to restrict itself absolutely to the interests of its clients, is possessed, as its international relations demonstrate, of a cold, calculating, and mathematical character, of an intellectual egoism far more stern than that of the

individual, because it is not tempered by the rational sensibility of the latter. History shows the excesses which may result from it when authority is centred in the hands of one man. If this axiom is no longer manifested in our democracies it is because society is in our day public property, the aggregate of its citizens, who, though scattered and segregated in infinite ways, watch it and prevent it from transcending the proper measure. What will our great syndical associations of individuals become in the future? Are they approaching to the type of social egoism, or to the type of individual egoism. May they not in certain circumstances, as in the case of strikes and workingmen's unions which embrace both hemispheres, become a menace at once to society and to the individual? Instead of contending with the individual who is still easily guided by sentiments and even by pretentious words, society will have to do battle with compact bodies of individuals who have but one dominating guide—the absolute necessity of its nameless and irresponsible members. I have been a close observer of their doings. They commit sometimes collectively and with calm deliberation monstrous acts of which their members individually would disapprove, for the responsibility falls on no one in particular. The most moral being, despite the picture which we have drawn of him, is the individual, and that for reasons which I shall give later. Associations are less moral. The state would be even less moral than associations, were it not for public opinion and the fear of revolutions. And why? Because the individual alone has a sensibility which at times neutralises egoism, whilst syndical associations have the same egoism without anything to offset it.

Among associations there are some which merit particular attention: commercial associations for aiding and succoring individuals and "mutual" associations for the same purpose. They are concerned on one hand with saving, and on the other with insuring the individual and his family against disease, loss of employment, accidents, and all the other unknown possibilities of the morrow. Saving and insurance are the expression of a quality, foresight, which some animal species possess in the highest degree and others not at all, which the lowest human species do not possess, which

among civilised men is more or less developed, and which people are unanimous in regarding as one of the characteristics of the Celtic race (the brachycephalic of western and central Europe). This quality is certainly one of those which are most physiological and contributes, consequently, the most towards the personal happiness of the individual. It flows from the idea that the existence of every person embraces three periods: one of preparation, one of work, and one of rest. And that in this last period where the physical and intellectual faculties are reduced in power, the first necessity is not to be dependent upon the care of any one, not to be left to the mercy of any of those numerous reverses from which the bravest and strongest are not exempt in the struggle for existence, and never to have recourse to private or public charity. It accords with the desire for stability and for the enjoyment of the fruits of life in the environment in which one is born, the enjoyment of a home, which is opposed diametrically to the spirit of Bohemian unrest which tends to become general in the closing days of our century and is the source of so many evils. Evidently society should look with favor upon the practice of saving, of acquiring annuities for life, *pensions de retraite*, upon the establishment of combinations for guaranteeing dowries to young women, competencies to young men beginning life, and provisions for widows and orphans. We say that the state owes protection to children, to the crippled for life, to all whose parents fail in their duty to them prior to the period when they are competent to manage their own affairs. But it really falls to the lot of associations for mutual aid to include within their sphere of action the care of children. They look now-a-days to the needs of adults, but they should also think of the needs of the children and the adolescent. The more the state shows itself to be intractable in the matter of compassion, the more these associations will develop in this direction as well as in others. M. Chevallier in the work cited above shows that the great extension of societies for mutual aid in England took place subsequently to the revised Paupers' Law of 1834, that this law rendered the workhouses generally detested, and that the workingman was in this way brought to the desire to protect himself. He shows

also that home assistance furnished by the state hindered the development of providential societies, all of which is a repetition of the truth that the state should encourage such things but should directly interfere as little as possible.

There is a quality inherent in the human race, almost the exact reverse of the preceding, which society should also favor, not for the interest of the individual as above, but for its own general interest. It is distinctly marked in the Anglo-Saxon races, and consists not in placing one's savings aside so as to be able to draw therefrom interest, dividends, or security for the morrow, but directly to cause them to multiply by more or less boldness. It is the spirit of enterprise, symbolised in the saying "go ahead." Its drawback sometimes is the accumulation of too great wealth in the same hands and thus the furnishing of a foundation for all the objections which are raised to-day against capitalism. Its advantage is the increasing of the circulation of wealth, the affording to it greater chances for distribution among the more active laborers and the producing of the means of existence and the objects of comfort in large quantities from which all cheaply profit. We will not insist upon this subject, which borders upon struggle, activity in general and its rewards, of which we have spoken sufficiently. There is no doubt but society should look upon all such efforts with favor and should encourage all initiative in directions which may give profit to all.

Another direction which should be encouraged is the development of the intellectual faculties, the preference for pleasures of a higher order and consequently the raising of the level of the human species more and more above that of other animals. We speak of the sciences, of arts and letters, and of their applications, whatever be their kind and degree. And this leads us to education.

Education has two objects. The first is to shape the character of the generation which is entering upon active life, to discover and to develop the aptitudes which children possess. We shall see later what is to be thought of the second object. In virtue of the principle that the state should not interfere in things which individuals are willing to do, education should be free. But its duty is

to encourage in that way all private efforts, to watch it carefully, and to give its sanction to the certificates and diplomas which issue from it. We have seen that the state should take charge of abandoned children whose parents refuse to prepare them for the struggle of the future. Whether given by the family, by private institutions, or by the state, directly or indirectly, it is at the start obligatory primary instruction. It should aim chiefly to fashion the cerebral organ, to inculcate common sense, spirit, habits of observation and logical induction, ready memory, etc. At the second stage come the secondary schools of a general character, the different professional and special schools, access to which as the result of an examination revealing the inclinations of the scholar will be made easy by the state to children whose parents are unable to defray the expenses of tuition and maintenance. In the third stage, that of superior instruction, there must be also entrance examinations and also free tuition and support. The difficulty is to make families comprehend the obligation resting upon them of giving to their children the maximum education of which they are capable. With public opinion and some few inducements and expedients, this is not impossible. Why, in our elections, should not two votes be given to persons holding diplomas from the secondary schools, and three votes to the graduates of institutions of the highest grade? Why are not certain diplomas obligatory for filling governmental and administrative positions? Is not politics itself a science? The aim is that no child should be deprived of the means which are capable of emphasising and developing his natural aptitudes. The principle is that the state, without interfering with the rights of the family when the latter fulfils its duties, nevertheless owes protection to childhood, as later it is committed to neutrality towards the individual entering the arena. By a progressive artificial selection of the kind indicated above, society would procure the best and greatest possible returns from its population, with whose prosperity it is entrusted. The higher individual variations would come to the front; the mean variations would be enabled to display themselves in the best and most appropriate conditions; the lower vari-

ations for which there is no hope, would alone be sacrificed, but the emulation of the struggle would greatly diminish their number.

The complement of this education would be laboratories for original research, public museums and collections, a few very special superior schools, model farms, national manufactories, and lectures which I might style luxuries, and which in the American phraseology are said "not to pay," but which are yet absolutely necessary for societies that are anxious to hold their own in the steeple-chase of progress. The state, if it does not take upon itself its duties directly, should at least carefully see to it that they are fulfilled.

* * *

The second object of education implies a broader signification. It looks to the public morals and to individual habits of feeling, thinking, and acting, independently of the useful or disadvantageous effects which they may have, and of the pressure exercised by the laws. It is concerned with the external conditions which are to be adapted to human nature, or with those aspects of human nature which are to be adapted to social conditions, and has for its direct object the intrinsic progress both of society and of the species. Two systems here confront us. In the one, evolution is considered as always ending, after oscillations for good or for evil, in the best possible result, and is consequently abandoned to itself, that is, to the free play of individualities and of circumstances. It is the *laissez aller*. In the other, evolution is considered as not giving desirable results and as requiring, therefore, guidance toward the end to be attained—the greatest happiness distributed among mankind in the most equitable manner. This is the system of interference.

And this brings us to the ego whose history we have traced in describing the individual at pages 562–566 of *The Monist* for July, 1898. In every individual, as we have endeavored to show, conduct is the outcome of three factors. The first is the ego which is inherent in the animal and exists in man as in all animals,—with this difference, that man having more intelligence, this ego assumes in him a high authority. It is the guide and guardian of the individual, it

has no object but the needs of the individual and their satisfaction, it is devoted entirely to these objects, it is egoism incarnate. This is the animal ego which we have portrayed in such sombre colors. The second factor is the product of habits of feeling, thinking, and acting as they are formed in ancestors and bequeathed to the individual in the shape of predispositions, which when confronted with conditions similar to those which have engendered them, are appropriately developed and have a weighty influence on the acts of the individual. This is the ancestral ego. The third is the product of the habits of the individual himself, acquired during infancy and the course of his life, depending on the maternal and primary education, on the comrades with whom he has associated, upon the examples which have been set him, upon the methods of feeling and thinking to which he has abandoned himself, upon the ideas which he has formed, and the allurements which they involve. This is the acquired individual ego. It also has a profound influence upon the acts of his life.

The animal ego reduced to itself is all-powerful in its activity, but its interference is not obligatory, as has already been said; it is optional; it is affected when the attention of the ego is sufficiently aroused and when the ego is resolved to have full sway. The two other egos, on the contrary, are passive. An excitation arrives at the cerebral centre, awakens these egos, and brings about the reflex action which it has already produced. Combined they constitute the unconscious ego which answers spontaneously to the demands of the individual when the real or conscious ego is not moved to intervene. The conduct of man, neglecting the purely medullary reflexes, is the outcome now of the one and now of the other. The conscious ego is the author of reasoned and directly willed acts, the unconscious ego is the source of instinctive and more or less spontaneous acts which are termed "impulses."

But the peripheral excitation which has reached the brain does not always directly awaken there the motor reaction; it also awakens the sentiments and ideas which hereditary habit and acquired individual habit have established in previous periods, together with the entire network of thought which is attached to it. Little as the

conscious ego occupies itself with what takes place in this labyrinth, still the sentiments and the ideas awakened, arouse of themselves the acts which are in habitual correlation with them, acts which even the conscious ego, if it were in full possession of itself, would probably never have committed. Thus a host of actions are explained, which society regards as proper or deserving, and which are yet in disaccord with the reasoned interests of the individual,—among them being acts of self-denial, generosity, and devotion.

Now of what are the ancestral ego and the individual ego which mutually strengthen each other, the outcome? Of modes of living and instruction, of impulses in ancestors and in the individual, which can be governed, evoked, and created. The animal ego knows but one thing—itsself, its interests, and its pleasures; the acquired ego acts as it has been in the habit of acting, and as it has been taught. The first calculates, the second obeys automatically. The first has its roots in the physiology of the organism and is incorrigible. The second can be moulded, adapted to social needs, and trained to feel and to think as the general welfare requires. The results of education, taken in its broadest sense, are brought to bear upon the acquired ego from the first generation, when it was formed, but more so upon those which follow where the same education is repeated, and where heredity comes to its assistance.

The ways and the means remain. In the first rank appears education by the family, its basis being respect for ancestors, veneration of their memory, and the meritorious examples which are to be cited from this source. The natural rôle of the mother is to form the heart, that of the father to shape the intellect by implanting in it the necessary notions of the reciprocal duties of men in society, of obedience to laws, of the responsibility of every one for his acts, of the obligation of every person to carve out his own destiny,—in fine everything which is indispensable to the existence of life in common. Upon this chapter of the family we should have much to say; we should have to recall all that we have seen of this subject among animals and at the dawn of human society. The problem of woman at the present day would be added. We should have to place in the foreground the Anglo-American movements

for her emancipation and the ideas which *The Monist* has somewhere characterised as French, regarding her rôle as a guardian of the domestic hearth, as a conservator of altruistic sentiments, and as the educator *par excellence* of children. We should have to ask which of these opposing evolutions are best qualified to lead humanity to happiness, and whether we should in our desires prefer the point of view of nature or the point of view of philanthropy. But this would require much space, and the subject deserves its own separate and full treatment.

In the second place comes the education which falls without the sphere of the family and is affected by the environment; that is, on the one hand by companions, examples, the conditions in which one lives, the allurements to which one is subjected; and on the other by the school, the books and the magazines which one spontaneously reads. It is undoubted that at the start primary instruction should not run counter to that of the family; that on the contrary it should strengthen it; that one should not make freethinkers of children prematurely; and that without touching the liberty of conscience, one should inculcate in them the necessary principles of the conduct to be pursued in society, which can be recapitulated in the axiom "Not to do unto others what we would not have them do unto us," and conversely. Of all the agents of education outside of the family and the school, the most active without doubt are the books and the journals which one takes, not for instruction but for distraction. But under what various aspects are these not presented? What wonderful services might they not accomplish in the hands of men who had the true sense and feeling for the beneficence that could be spread by them. They could habituate people to sound and comforting ideas; they could set the example of the morals which it should be desirous to establish; they could elevate the heart and the mind and facilitate the task of the wise, who see afar. But in general their action is the reverse. I dare not say what the state of affairs in the United States is in this regard, but here where I am writing, the picture is a sad one. The good is eclipsed by the evil; the liberty of writing and of publishing is one of the conquests of modern times, but in the stage which it has

now reached it is merely an unnamable licence. The most shameful novels, which show the human species only in its basest aspects, and which glorify vice, are in all hands, and especially among the lower classes to whom they are furnished for a mere nothing. With certain reviews and journals they contribute more than any other cause to the increase of the number of criminals, and especially of young criminals. In order to sell, these journals shrink before nothing; they exalt the passions, openly cultivate scandal, preach insubordination, and crush the holiest and most useful sentiments under foot. The press should be the great educator; it is the great demoraliser. If any example of the contradiction between the principles, or rather the desires, and reality is conspicuous, it is assuredly here. On the one hand it is desired that the individual should enjoy all his liberties, although the very essence of life in common is the restriction of those liberties. On the other hand people seem to regard it as their duty to furnish the proof that unlimited liberty is impossible. There is no middle course. The press, the novel writers, and the pamphleteers must understand that their mission is to encourage the development of the necessary morals, or that they must be prepared for being repressed.

In the third instance, come the laws and institutions which best foster the customs and habits which it is desirous to develop, and the modes of feeling and thinking which it is desirous to arouse. The state should be strict with those under its care, but also strict with itself, and should give the first example of the virtues which it exacts. The individual responsibility of each of its employees, whatever their rank, should be absolute for every undertaking, for every infraction of the prescribed forms. The slightest failing on the part of the state throws trouble into the souls of individuals and authorises them to revolt. Everywhere, in the bureaux, in the tribunals, in its diplomatic service, it should be impeccable. But so long as war persists with its perverted ethics, there can be no hope of an absolute transformation of the public mind. The numerous and flagrant mistakes which are sometimes committed in the name of justice and for reasons of state, which authorise everything and anything, have pernicious results.

In the last instance come the efforts of private persons and of associations which are animated by a profound love of humanity,—the efforts of practical philanthropists, of philosophers striving to elaborate systems of conduct, and of scientists coldly analysing the difficulties of the problem. Here is the place to ask whether in order to give unity to all these efforts, the time is not ripe for establishing a code of morals concerning certain indispensable points regarding which the whole world is in accord.

We have seen how little a man amounts to in time and in space, in the hands of an irresistible nature which crushes him despite the fact that he has found a way to adapt some of her forces to his needs; how intensely he desires to live as fully and agreeably as possible while perpetuating his species, although as an individual his foresight reaches hardly beyond his children and grandchildren. We have seen, on the other hand, that society has adopted for its controlling principles not absolute truths but relative and necessary truths in order to fulfil the end for which it exists, and to enable individuals to live wisely and conformably to their desires; that among the principles of solidarity, liberty, equality, fraternity, and justice, none of them can withstand rigorous examination. Society is a solidarity of interests, and not a real solidarity; the basis of society is the restriction of liberty; equality does not exist among men nor in the results of their conduct; fraternity is but disguised egoism. In social practice these principles amount to this—solidarity, but psychical only; equality, but only before the law; fraternity, but only as a dream. Yet one of them dominates all the others,—justice, an imitation of relative and individual justice, and the synonym of “giving to every one what is his due.” Solidarity, in fact, implies justice; the restriction of liberty implies justice; justice implies equality; without justice there is no fraternity. Justice thus becomes the primal necessity *par excellence*, the postulate¹ of any system of life in common. Justice is our supreme desire, the ideal of which we dream in spite of all proof to the contrary, that

¹ “Any truth is called a postulate, which although not rigorously demonstrable must yet become practically admitted because of the necessity of its consequences.” —Paul Janet, *op. cit.*

which we say must be, that which we are bound to create and establish in spite of all, and in the face of all, of which we must be convinced and which must be taken as an article of faith.

It is justice therefore with which the necessary moral code which is to be enacted must in the first instance be saturated,—the commandments of society which are to be prescribed for the family and for the schools, notably for the primary schools.¹

But, it will be said, this code of morals and these principles, these habits or instincts impressed upon the unconscious ego, consolidated with time and sanctioned by the punishment which the laws prescribe,—will they be sufficient to assure in all circumstances the conduct desired? Should there not be sought in the individual organism itself, in its cerebral system, some influence which would act from this side on the unconscious ego and move it in the direction which society deems best,—in the direction of what is called the good?

The first influence to be invoked would be the categorical imperative of Kant. And without a thought of this and by ways which Kant would doubtless have rejected, it is precisely to this idea that we ultimately come. The individual feels with the sentiments and the ideas of his ancestors. These ideas deserve the qualification of innate. He acts with the habits which these ancestors have handed down to him and which education has confirmed. His animal ego reflects what the acquired ego has gained; he no longer knows whether he acts automatically or by his own initiative, and he adopts the good or evil as society wishes.

The second is that expounded by M. Guyau in his different works,² and which I shall paraphrase as follows: life which has reached the last stage of its evolution in the organic series, which has arrived at the point where it is aware of itself (consciousness), where it admires itself and everything about it (the æsthetic sense),

¹ See Paul Janet, *Éléments de morale pratique. Enseignement secondaire moderne conforme aux programmes officiels de 1891*. Paris, 1897. I take exception to Chapter X. only.

² M. Guyau, *Esquisse d'une morale sans obligation ni sanction*, Paris, 1892; *L'irreligion de l'avenir*, Paris, 1896; *Vers d'un philosophe*, Paris, 1896, etc.

where it diffuses itself over others (the moral or altruistic social sense), even over ideal beings (the religious sense). "Life, the most extensive and intensive possible, conscious of its fecundity," he says somewhere. "To live the maximum of life," he says again, "in the most varied manner possible and to cause that life to overflow upon others, is the end and the cause of our acts, and not the pleasure which we derive from them." It is the need of activity inherent in every organ, in every organism, and especially in the brain, of which I have frequently spoken, but the consecration of which, I add in opposition to Guyau, lies in the pleasure which this activity offers of itself. The objection is this. It is a power of expansion and not a guide to conduct in a determinate sense, useful to all. It is perfect in people like Guyau, a poet and a philosopher, who find happiness in the exercise of their highest intellectual faculties, but it is inefficacious in that other class of persons, and these are the great majority, who place their ideal in satisfactions of a different order. On this theory one can be a villain, a Napoleon, or a Rothschild.

The third influence is self-respect, human dignity, belief in one's superiority,—in a word, pride. Man, recognising his dependence, proudly and haughtily refuses to accept as the judge of his conduct any one but himself. This is stoicism in its general form. It is excellent for inspiring courage and for enduring undeserved adversity, but it is insufficient to arouse that generosity and tolerance which are factors of the conduct desired by society.

The fourth, which is derived from the two preceding, is the will which flows from liberty as it is understood by M. A. Fouillée. I shall recapitulate its main terms: "It is a characteristic of man that he is moved not by purely physical forces or blind instincts, but by ideas." "Ideas are forces which influence our conduct by the very fact of their conception." They are at once the cause and the end. "The evolution of nature can have no preconceived end, in the proper sense of the word, but the evolution of humanity has one, from the fact that humanity actually sets itself an aim, and imposes upon itself an ideal to be realised." "The idea of a society adopting liberty, equality, and fraternity as its end is the

highest moral ideal." To will is to be able. "Ideal liberty is a power of indefinite development, the essence of which consists in the power to throw off selfishness and to love, and the progressive realisation of which would lead to moral and social union among living beings." I confine myself to two remarks. Ideas conceived as moving springs of conduct are precisely those spontaneous impulses which I desire to create by education and heredity. Liberty implies the power of showing oneself unselfish, but in the same measure the power of considering everything in the light of one's interests.

The fifth influence is the prudent interest of Bentham and John Stuart Mill, by virtue of which the individual through careful reasoning identifies his personal welfare with the welfare of all. It is virtually the end to which the system that I have developed tends, save that I would replace the words "by careful reasoning" by the word "unconsciously." In fact, intelligence varies. Secondly, intelligence may in many circumstances, and precisely in those in which the unreflecting impulse is the most necessary, come to the conclusion that the interest of the individual is opposed to the social interest. In my system, on the other hand, the individual acts unconsciously in the direction required, for the simple reason that he has the habit of so doing.

A sixth influence is that physiological property on which we have so often insisted, which is highly developed in the majority of animals, particularly in herbivorous and domestic animals, and not less developed in man in the state of nature before the struggle with his fellow-beings broke forth and had not assumed in society so threatening a form. We are speaking of that species of cerebral sensibility which moves both man and animals to seek the company of their congeners, to derive satisfaction from their mutual relations, to love others, and to desire to be loved by others. It is altruism, of which the first stage is kindness and the last devotion; the most powerful physiological impulse next to egoism, although it is only an indirect form of egoism. To love and to be loved, next to eating, drinking, and acting is the first need of children. It remains intense to the age of puberty, and continues to the day

when the individual enters into the arena of serious life. In the old man who has no longer any of the cares of existence, it resumes its rights and spreads over his grandchildren. In the adult, in the moments of respite which the struggle leaves him, it is his repose, refuge, and recompense. How sad life would be without friendships! In the bosom of his family the wife satisfies the needs of the heart rather than those of the senses. The husband, who is less faithful in the second regard, is loyal as to the first. Man undoubtedly domesticated the dog by altruism, and every day we see him creating bonds of attachment to himself in the most different kind of animals, by simply asking for reciprocity. Altruism is the first source of sociability, as we have already demonstrated, and it is its consecration under its multiple forms of kindness, indulgence, tolerance, self-denial, sympathy, charity, generosity, devotion. This is the reason why, in spite of all the objections which may be raised to assisting the unfortunate, no voice is ever raised against it, and that there is no difference of opinion except as to the means. It is the only physiological force which can check in the organism itself the impulses of egoism and the many secondary forms which egoism assumes. To adopt reason as the instrument for combating personal interest after the fashion of Bentham, is to exaggerate its power. To adopt human vanity, liberty, or fear, is still more exaggerating it. The system of Guyau accords best with that of altruism, for to live in the happiness of others as much as in one's own, to exchange impressions, sentiments, and thoughts, is to live with greater fulness, and to see about one nature in all its smiles and beauties. Justice is a necessary regulator of social life; external equality which society offers is its corollary. The maximum possible liberty is the individual principle that comes next, and let us add to the latter, the principle of fraternity formulated by the Master, "Love ye one another."

With these two elements, altruism as the basis, and habits and social instincts as the means of execution, the desired end will be attained. The conditions to be sought, the reforms to be made, the new things to be added, are they not precisely what we see has been spontaneously effected in our best and most esteemed fam-

ilies? What, in the last instance, are the best of us, individually, if not the product of the transmission of virtues which our ancestors have bequeathed to us, despite our tendency to create other habits for ourselves, to wrest ourselves from the bonds of heredity, and to build up in ourselves independent originality? The good instincts which we may have, do we obtain them from the spirit of the century, from the cold reasoning of the day, which analyses the motives and the effects of all acts, and mathematically calculates its interests? No, we receive them from our predecessors. We are honest, proper, and loving because our fathers and grandfathers were so. Otherwise, how can the naturalist and the freethinker explain the flagrant contradiction which exists between his conduct and his reasoning. He sees only brute reality, he establishes the sad truth, he deduces the consequences of it, and yet he is unable to free himself from the most generous aspirations of his altruism. He places friendship in the front rank and practises it. Why? Because the soul of his ancestors is perpetuated in him, because he is their continuation. Yves Guyot, who professes egoism as the sole principle of individual conduct in society, writes as follows: "When I see a child beaten, and hear it cry, when I see a woman weeping, when I am the witness of suffering, I am divided into two persons. Another ego feels these pains. . . . All my fibres are set in vibration; the old blood of the soldier, the corsair, the hunter, which runs in my veins, seethes within me. . . . My instincts impel me to act." He speaks truly. It is no longer the egoist who is talking, but the altruist by heredity.

The establishing, or re-establishing, of the customs which are best adapted to social happiness and their progressive consolidation by heredity; the ego, without name, acting automatically in the direction which society deems to be the best; the individual and society, shaped by man as he shapes a plant or an animal conformably to his needs, justice as the regulator, and love as the ideal: such, in fine, is our system.

Is it necessary to add to this a grain of mysticism—a belief in the absolute, a belief in the individual surviving the body and preserving its memory? Or the transformation of the categorical im-

perative in the form which we have stated it, into a metaphysical entity? We do not believe that the system would gain by such an addition. It would be a dogma simply. It is true that justice, as we have shown in the case of society, deserves in some measure this name, and that liberty is not much farther removed from it. The essential thing is to attain the end, the greatest possible happiness. But is not illusion frequently happiness? Is it not often more beautiful and more consoling than reality? Read the heart-breaking pages of Guyau,¹ dying and still not abandoning hope. Would it not have been cruel to deprive him of it?

Certainly, but to admit illusion, even in the extremest case, would be tantamount to declaring that truth is insufficient, that there is no remedy whatever, and that human reason culminates in lamentable fallibility. But we have not reached this point. Truth, when we look it calmly and stoically in the face, is not so discouraging. Herbert Spencer, who concludes as we do as regards the necessity of developing altruism and certain hereditary habits, is wrong in his expressions of despair at the close of his monumental work. We differ from him in the point that he counts upon the free play of individuals and natural evolution, whereas we believe it indispensable that man should direct his own evolution. We, too, have had our moments of doubt,—not regarding the efficacy of our system, but regarding the possibility of realising it without the intervention of too much authority; but we have taken fresh confidence. We believe unqualifiedly in the great power of heredity, habit, and unconscious impulse over our daily acts. We are convinced that if society so desires it and comports itself properly, it can in a few generations transform sentiments and manners, and adapt them to its needs. The useful instincts have sprung up of themselves in animals. Why may they not, with the assistance of reason, be created in man? Speaking only of France, I have already seen about me for the last ten years, certain indications pointing to renovation; new social classes will achieve success where the old have failed. No doubt there will be storms, good

¹ Pp. 26-28 of his *Esquisse d'une morale*.

and evil alternations, but in the end evolution, which proceeds only by oscillations and starts, will enter on the right path. Let us not despair. Man is too powerful to fail in reaching his end. The golden age of humanity is ahead of us, the sun of the twentieth century will be that of truth. "Error is a Penelope who, without wishing to do so, is incessantly unravelling the texture which she has woven. Truth, on the other hand, in the struggle of ideas for life, will sooner or later bear off the victory." (Fouillée.)¹

Let us revert, as we near the close, to the question of the functions of the state,—first the essential functions which it cannot possibly cast off, and secondly the facultative functions, including one of the highest importance, that which concerns progress, or more exactly the best adaptation of things to men and of men to things. It may be summed up as follows:

The state is responsible for the existence of society without and within. To this end it is armed with all powers and uses them

¹ The article of Professor Dewey in *The Monist* for April, 1898, and that of Dr. Paul Carus in *The Monist* for April, 1894, on the *Evolution and Ethics* of the late lamented Professor Huxley, have just drawn my attention to Vol. IX. of the *Collected Essays* of this author. I was much struck with the identity of my conclusions with those of Professor Huxley, published in 1888, 1893, and 1894. I am not astonished at the fact, however; for, proceeding by the same methods, and with the same facts and in the same spirit, we ought necessarily to have reached the same result. I call attention to some few of the propositions.

"Social progress means a checking of the cosmic process at every step, and the substitution for it of another which may be called the ethical process."

The science of ethics or morals is that of the best conduct for the individual and society. The morally good is what answers best to the general good of the community, all other things being equal.

Social progress is effected, not by self-assertion (my "free expansion of life" in the individual, Guyau's "need of living at the maximum"), but by self-restraint and self-discipline.

"The intelligence which converted the brother of the wolf into a faithful guardian of the flock ought to be able to do something toward curbing the instincts of savagery in civilised man."

Huxley does not formally indicate the ethical process which I set up; namely, the moulding of the acquired and unconscious ego to conform to the needs of society; but it follows implicitly from numerous passages of his on habits, reflex actions, heredity, etc. We find, in fact, that there is no choice; either we have to abandon ourselves to the *laissez faire*, which is nothing but the cosmic process itself and can only lead to anarchy and the rule of the strongest; or, we must, by taking our stand on the nature of man, direct the ethical process, as I have explained.

as it sees necessary. It is entrusted, further, with its prosperity, present and future, and its guidance is limited here by the laws, which it is as much bound to obey as private persons are. These laws change with the legislature, and the question recurs: What is the scope of power that should be accorded to it? Should they be augmented or curtailed? Should more be given to the state and less to the individual, or conversely?

It is here that the difference of opinion of statesmen, economists, and sociologists appear. There are extremists on both sides. On the one hand are the collectivists who wish to lodge every possible power in the state, to revert to the communal or national form of property existing in the majority of primitive societies, to regulate the entire current of life, to give to each according to his needs strictly considered, and not according to his labors,—in short, to suppress individual responsibility. On the other side there are the anarchists who refuse to consider the least restriction of natural liberty, who attack thus the very principles of society and go so far as to say that wherever three men are assembled there is a tyrant. Neither the one nor the other of these systems deserves to be discussed. It is certain that the time has come, that there are many reforms to be made, that all have not their equal share of the means for administering to their needs and for becoming established in life, but the difficulties cannot be solved by exaggerations which are at downright variance with practice. Between the two extremes are the advocates of authority who believe in a strong state thoroughly centralised, but a state which grants to the individual sufficient liberty to enable him to move freely in the sphere of his personal affairs; while there are also the radicals who are for decentralisation, who would give the maximum of liberty to the individual without going to the extreme of the anarchist, but who are too hasty to be practical.

In the centre are the progressivists whose name is perfect and who also deserve the name of opportunists, as they are called in France. For us they are the sages of Plato, those who know how to put to use the teachings of social science according as it is de-

veloped, those upon whom I would count for directing the social evolution in the direction and by the means which I have sketched.

It is from social science, the most important of the applications of anthropology, of which sociology is a branch, that all light is destined to come. Born of yesterday, it already bears testimony to its sweeping influence. Its program is clear: to classify the ends in view; to look the difficulties courageously in the face even where unsurmountable; to establish principles; to seek to reconcile the contradictions which we have instanced between the conceptions and desires of man and the realities of nature; to suffer every progress to come to its maturity; to proceed without prejudice, without theory, with a full knowledge that the absolute good cannot be realised, but only a relative and progressive better. The developments which we have been following in this long work reveal our tendencies at the points where we have not indicated them. For us, the individual, the family, and personal property are the social tripod. For us, the political formula is as follows: the maximum possible to the individual, the minimum possible to the state, and in the latter the most possible to the local authorities, the least possible but the necessary to the central authorities. If I am not mistaken this is the condition that exists in the United States.

* * *

We have reached the conclusion of our long work which we had entitled "Science and Faith." We have spoken much of the one and very little of the other. The reason is that the two mutually exclude each other. Science is knowledge; faith is belief. Science considers things objectively and accepts only what is demonstrated by observations, *perpendæ et numerandæ*, and by generalisations and inductions which go with it, stopping at agnosticism.¹ Faith, on the contrary, is subjective, individual, and dependent on cerebral sensibility, as the latter has been constituted by heredity, education, habits, and temperament of the subject. Orators, who like the celebrated Dominican, Père Didon, seek to demonstrate

¹ That is, stopping where the facts abandon us, not having recourse to a world where no positive and objective facts are forthcoming.

the compatibility of the truths established by science and the beliefs dictated by faith, only shatter the latter; a faith which is examined and shown to be in accord with facts ceases to be faith. It is warrantable that in the epoch of humanity in which we are at present, there is utility in extolling certain articles of faith, as Kant has done. It is quite warrantable that certain philosophical doctrines are advocated; and one cannot admire too much the sages who thus devote themselves to the mission of work for humanity. I am not far even from admitting that the four or five principles, especially justice, which society takes for its base and ideal, should be converted into articles of faith, but I would have it perfectly understood that the two domains of science and faith are totally different—are two contrary poles.¹

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¹ I have been much struck with the religiosity, excessive but perhaps necessary in its excess, of the English and the North Americans, joined to a calculating, free, and well-balanced psychological state, which I appreciate all the more highly because if I am not mistaken it is the same that governs my own ideas. This religiosity was principally formed some *centuries ago* with the Puritans and the Presbyterians of Scotland. Nevertheless, as an anthropologist, I believe that this placid religiosity, which is so different from that of the dark populations, goes back to very remote times and is one of the distinctive traits of the blond races. For proof, witness the religion of the Druids, of the prehistoric Gauls (I speak of the tall, dolichocephalic and blond Gauls, and not of the brachycephalic Celts), the congeners of the Cambrians, British, Danes, and so forth.

appeared, as in the curve, every point of it is above AO , and increasing, so as we proceed from left to right. But the development has been very slow—the direction of the curve for ages seems to be almost parallel with the course of time itself. When we reach, for instance, the highest form of organic life of the period it will be very low.

AN ILLUSTRATION.

A MATHEMATICAL curve sometimes serves to illustrate the results of investigation in other sciences. Here, for instance (Fig. 1), is a part of a plane curve of the second degree—an hyperbola—referred to its rectangular asymptotes.

May not this curve be used to illustrate the theory of the evolution of organic life upon our planet, from its lowest to its highest possible forms?

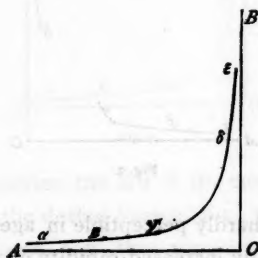


Fig. 1.

Suppose the direction AO to represent the duration of time, and the direction OB to measure the degrees of organic development through the possible range of organic life. Then the curve ae , combining the two directions according to a certain law, may represent the successive development of organic life during the course of time.

Thus, organic life of the lowest possible form is supposed to have begun on the line OA at an unmeasurable distance to the left of O . From that point on, in the course of time, improvement has been continuous. In each successive age, higher forms of life have

appeared, as, in the curve, every point of it is above AO , and increasingly so as we proceed from left to right. But the development has been very slow,—the direction of the curve for ages seems to be almost parallel with the course of time itself. When we reach α , for instance, the highest form of organic life of the period is still very low.

Suppose now that the distance from the line AO to the point α represents the degree of development attained by a trilobite, and in like manner think of β as representing a fish, γ an anthropoid ape, δ the lowest known type of man, and ϵ the highest known type of man. Then it will be interesting to note the *direction* of the curve at each of these points. At α , and even at γ , it is almost horizontal, and if development were to continue at the same rate, im-

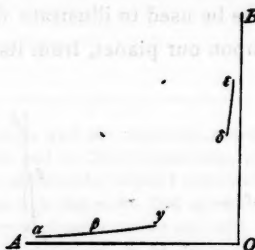


Fig. 2.

provement would be hardly perceptible in ages. How different at δ and at ϵ ! The greatly increased rapidity of the improvement results in giving the curve an entirely new direction. Heretofore, we may say, the development has been mainly physical; now it appears to be mainly psychical—intellectual, moral, spriritual—reaching heavenward, if we may think so. Yet the curve, from end to end, even in the beautiful deflection from *onward* to *upward*, is following its mathematical equation. Every point in it satisfies that law, and we are reminded that the theory of evolution also embodies a law.

But suppose, now, that instead of Fig. 1 with its continuous curve, we had only Fig. 2.

To one not familiar with mathematical curves, there is little to

suggest that these two bits of line, $\alpha\gamma$ and $\delta\epsilon$, have anything to do with one another. Their directions are quite different, and they *seem* to be straight rather than curved. A mathematician, however, might look at them in a different way. He might turn the page up edgewise and squint along the lines, and he would then find that $\alpha\gamma$, and even $\delta\epsilon$, show a perceptible, though slight, curvature. He would apply such micrometric tests as he could, and from the measurements obtained would carefully calculate the equation of each short curve, and the resulting equations would be found identical. One conclusion only is possible,—the two bits of seemingly straight line are parts of *one curve*, and he proceeds to reconstruct the missing parts as far as ever he will, with mathematical pre-

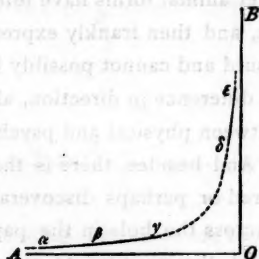


Fig. 3.

cision, by simply applying the law of the curve. This reconstruction may be seen in the dotted lines of Fig. 3.

But let us suppose, now, that some one had torn out a piece of the paper between γ and δ , and so had rendered it impossible to reconstruct the curve across that gap. Would our mathematical friend be any the less sure that the two bits of curve are in reality the same curve? No. He has already discovered that they have the same equation, involving the same variables in the second degree, the same constants. Proof does not depend upon ocular demonstration, nor does it consist in ability to mechanically reconstruct all the missing parts.

But how is it in the matter we are trying to illustrate? Another friend of ours, a theologian, has read Darwin and Wallace and Milvart and Romanes, and is informed and believes that organic life

in the lower forms of existence has been developing according to an ascertained law. So far, however, the direction of the development appears to be almost on the dead level of animalism, though some semblances of intelligence are found among the higher animals. He has found man also developing at a wonderful rate in a direction quite different from that other, that is, intellectually and spiritually. In other words, reverting to our illustration, he has discovered the two bits of curved line shown in Fig. 2 and he has noticed the hypothesis that they are parts of the same curve and have the same equation. He agrees with a recent writer that natural law extends into the spiritual world, but when brought face to face with the proposition that the development of man and the development of the lower animal forms have followed the same identical law, he hesitates, and then frankly expresses his opinion that the proposition is absurd and cannot possibly be true. Why? Oh, there is that marked difference in direction, almost the whole quadrant of difference between physical and psychical, or between material and spiritual. And besides, there is that great gap, with no missing link discovered or perhaps discoverable. He cannot reconstruct the curve across the hole in the paper. Ocular demonstration is not to be had, though it would seem so desirable, not to say essential to proof, in a proposition of this kind. And who knows, after all, whether the micrometric tests which Mr. Darwin and others applied were not so minutely *in*-accurate as to be practically worthless,—whether the calculations based on them have not been vitiated by some undiscovered error?

Moreover, our theological friend has followed some observations long since made on the bit of curve called $\delta\epsilon$, and has held a theory of man somewhat different from our more modern and crude scientific one. He believes that $\delta\epsilon$ is a curve, sure enough, but he has been taught and has been teaching that its nature is parabolic, or perhaps, even elliptical, rather than hyperbolic. He has an equation which seems to fit it excellently, and has himself done some reconstructing in accordance therewith, as in Fig 4, whereby it appears that his curve does not start with a point at an unmeasurable distance to the left, and lower down than a polyp,

but really began only a little lower than a certain Θ , at a' ,—Adam, the first man. Man had to fall before he could begin to rise. Paradise must first have been *lost* before Paradise can be regained. How else is it possible to account for the phenomena of *Sin*?

A simple illustration should not be pressed too far. Of course, it does not prove anything. But this one suggests a question or two. In Fig. 4 the dotted curve has been constructed from observations made on one small part of one leg of it. The hyperbolic curve in the preceding figure was constructed from observations made on parts of *both* legs. The data in one case may be said to

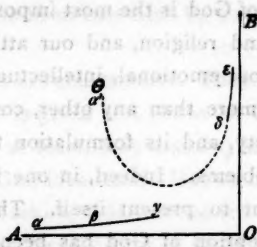


Fig. 4.

be all spiritual. In the other case, there are two sets of data. Which process would seem more likely to lead to a correct result? And again, it is not hard to believe that one of God's laws, in the form of a mathematical equation involving the squares of X and Y , represents the very nature of an interesting curve, of whose legs the one is almost horizontal, the other almost perpendicular. Is it really any harder to believe that under another of God's laws, involving variation and selection, the evolution of organic life has successively produced such forms as the longitudinal worm, the half-upright ape, and the upright man?

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CHICAGO.

but really began only a little lower than a certain θ , at $\theta = \theta_0$.—Again, the first man, Man had to fall before he could begin to rise. The adjustment first have been, and before it can be regained. However, it is possible to account for the phenomenon of θ_0 . A simple illustration should not be pressed too far. It cannot do not prove anything. But this one suggests a question. Two curves, the lower curve has been constructed from observations made on one small part of it. The hypothesis curve in the preceding figure was constructed from observations made on a large part of the curve. The data in one case may be said to

GOD.

THE conception of God is the most important idea of philosophy, science, and religion, and our attitude toward it is of vital importance for our emotional, intellectual, and moral life. It is a thought which, more than any other, covers the unity of existence in its entirety, and its formulation touches upon a great number of other problems. Indeed, in one form or another it is likely at any moment to present itself. Thus it is a matter of course that the conception of God has been approached in various ways and can be treated in the most diverse manners. We may with mystics abandon any attempt at comprehending the problem and indulge in purely intuitional contemplations, which naturally will assume the form of visions and ecstasies. We may with moralists point out the close relation between God and duty and preach the sermon of the categorical imperative; or we may with the scientist seek the ultimate *raison d'être* of creation and trace the eternal, the everlasting, the permanent in the transiency of existence. We might combine the three methods and start from the needs of these three aspects of human nature, the head, the heart, and the hand, and proceed on these three avenues of our life to their centre, in the hope of harmonising the results of our methods and reconciling apparent contradictions.

Yet we may take still another road which is very promising. The God-idea is of historical growth; it is (in the form in which it exists in the minds of the present generation) the product of a long evolution; it represents aspirations definite in kind and tending in a definite direction. These aspirations are by no means

all consistent; to a great extent they are conflicting and even directly contradictory. Many of them are conservative and reactionary; others progressive and radical. A great part of them partake of the nature of instincts. They are, in their ultimate constructions, submerged in the realm of subconscious and unconscious soul-life. In other words, they are based upon arguments which do not appear on the surface of conscious life but are buried in the traditions of the past, and have originated under the influence of the experiences of our ancestors from time immemorial, still embodying the notions of primeval man, which, however, have been added to and have also been corrected by considerations of a more matured period.

All these methods are constructive. They are methods of handling the material that is given and (however critical we may be in details) assumes (or, at least, may assume) as a matter of course the legitimacy of the God-idea itself. But we might attack the subject in quite another fashion, a fashion which at first sight appears to invalidate the whole issue, but may prove after all most fruitful by assuming an attitude of doubt and subjecting the God-idea to a critical analysis.

What if, after all, the atheist be right? Would not the whole question as to the nature of God become irrelevant? Would not the visions of the mystic have to be regarded as aberrations of the human mind? Would not the God-idea in science and philosophy be out of place, and had not ethics better dispense with it as an unfounded hypothesis, while in history it should be treated under the heading of superstitions?

These questions we venture to answer with a decided no. Even from the standpoint of the atheist, the God-idea remains the most important thought in the history of the world. It is neither irrelevant nor an aberration, but contains the most important, the deepest and most comprehensive, philosophically the most explanatory, and practically the most applicable truth of all truths,—a truth which is expressed in a most telling popular way, setting forth its main features in striking human analogies and with a directness that shows at once the practical and personal interrelation of the

unity of the whole to all parts, of the universal to all particulars, of God to every one of us.

There is, so far as I can see, only one more attitude left, which is the agnostic position, holding that we cannot know whether God exists or not. Agnosticism, which may briefly be characterised as a bankruptcy of thought, is not only the weakest but also the most injurious philosophy. It is the philosophy of indolence which on account of its own insolvency declares that the most vital questions of man's life, the questions of the soul, the soul's relation to the body, the immortality of the soul, the existence of God, the creation, and the ultimate purpose of being, are beyond the reach of reason. The agnostic argument consists in glittering phrases such as "the finite cannot comprehend the infinite," which are unmeaning, if analysed, but, as a rule, strongly appeal to the Philistine who is satisfied with mere words.

"The finite cannot comprehend the infinite" is such a common-place expression and is so thoughtlessly repeated by both reverent and irreverent agnostics, that I may be pardoned for a short digression in pointing out its weakness. What does the phrase mean? Are the terms "finite" and "infinite" used in their strict scientific, i. e., their mathematical sense? Apparently not. For in mathematical language the "infinite" as such is not less definite and clear than the "finite." It is a process unlimited, while the finite is limited. A mathematical line is infinite. The decimal fraction 0.333 , etc., if it were actually extended to equal one third would be an infinite series; the tangent of 90° is infinite, etc., etc. All these infinitudes are as little incomprehensible as the finite numbers 1, 2, 3, or any other magnitude. Are the terms finite and infinite used in the mechanical sense? Apparently not. An infinite chain, an infinite screw are mechanical contrivances which serve the same purpose over and over again. There is no beginning and no end; but an uninterrupted round of revolutions. A circle, a ring, a wheel, capable of unlimited functions by returning again and again to the starting-point, are not more incomprehensible than things definitely limited in their work, having characteristic starting-points, progressive developments, and final consum-

mations. There is as little sense in the saying "The finite cannot comprehend the infinite" as in the words "The rational cannot comprehend the irrational," or "The wise can never comprehend the unwise." If, however, the words "finite" and "infinite" have a special sense, the inventor of the argument should first define the terms before he expects us to accept his conclusion as valid.

There are two kinds of agnosticism: the pious agnosticism of him who would not allow the light of science to shine upon the problems of religion; and the infidel agnosticism of the scoffer who argues that, knowledge on matters of religion being unobtainable, we ought to leave religion alone. Both views are equally reactionary; yet at the same time both are equally acceptable to the Philistine who loves stagnancy because he dislikes to do any thinking for himself. While even the atheist's denial will be helpful, the agnostic position is neither theoretically valid nor practicable, for it leaves all opinions, be they scientific, superstitious, or mere guesswork, on the same level of equal incommensurability. And this verdict holds good for agnosticism in all forms, also for Mr. Spencer's popular agnosticism adapted to the demands of the average reader, and even for Kant's idealism which is both deeper and more dignified. Kant demonstrates in his *Critique of Pure Reason* the fallacies of the God-idea and the impossibility of offering any positive proof in its favor; yet he proposes in his *Critique of Practical Reason* to postulate the existence of God. If God is to be of any account at all, his existence must not belong to things hypothetical. A God whose existence has to be postulated is worse than no God at all, and even atheism is preferable to that undefined theology which rests its ultimate argument upon our utter ignorance of things supersensible.

If we wish to be clear on this subject, which has been surrounded with the clouds of dust raised by the quarrels of schools and factions, we must trace God in the facts of our experience. If he is not there, he is nowhere; if he is only beyond the clouds or in the realms of metaphysics, his existence is of no account and we might as well do without him.

The monotheistic God-idea is very old. Centuries before the prophets of Israel purified the Jahveh cult, there were philosophers in Egypt and priests in Mesopotamia who proclaimed the supremacy of the one God. An ancient brick found at Ur, whose date has been ascertained to be 2500 B. C., contained the following litany for temple service:

"In Heaven who is supreme?—Thou alone art supreme.

On earth who is supreme?—Thou alone art supreme.

The word is proclaimed in Heaven,

And the angels bow their faces down."

And a hymn of about the same date reads:

"Long-suffering Father, full of forgiveness,

Whose hand upholds the lives of mankind,

Lord, thy deity is as wide as the Heavens

And it fills the sea with awe."

In the history of the nations Israel has become the prophet of this monotheism. We can still trace in the Old Testament the process of purification. The tribal deity of Jahveh, worshipped under the symbol of a bull in the national sacred cities of Bethel and Dan, gradually changed into the universal God of justice and truth, until in the early Christian era he was conceived as the triune Father, Son, and Holy Ghost,—which in philosophical language means God is *Grund*, *Ursache*, and *Zweck* of existence. He is (1) the *raison d'être*, or law of being, (2) the evolution of life in its onward aspiration, and (3) the ideal and its final consummation, the aim and purpose of existence.

Now, we must in all religious ideas distinguish between the symbol and its significance, the myth and its meaning, the allegory and its lesson. Accordingly, we must analyse the God-idea and distinguish between those features which can be retained as literally true and those which are allegorical. Literally true are those features alone which can be traced in the facts of experience and established on good evidence as unequivocal truth.

In the mythology of monotheism God is called the Father, the Lord, the Creator, the Judge. What can these terms mean? Have we to understand that God is a man as we are, a parent who pro-

creates children that grow up and develop in his own image? Shall we regard Him as a king after the fashion of earthly rulers, surrounded by angels and archangels, as our sovereigns are by ministers of state and secretaries? Shall we believe in the judgment-day as pictured by Michael Angelo and other Christian artists? The atheist rejects the belief in God on account of the crudities of the myth if literally accepted. And he would be right if these crudities were the essential features of the God-idea.

The child-like theist says: "The world is governed by a good Father." The atheist says: "The world is governed by law." Both are wrong. There is no governing at all; the term governing is a pure allegory which in its literal significance does not apply to the processes of nature. The truth is, there are uniformities of nature which can be classified in universal formulas describing all possible happenings of a special type. Thus the law of gravitation does not govern the motion of falling bodies and of the coursing planets, meteors, and suns. The law, so called, is a descriptive formula which states in the tersest way possible the mode of action which things of a definite quality will take under certain conditions. That which makes the stone fall is the stone's gravity, which is an attribute of its mass, and the action of the stone's gravity depends upon the stone's position in the universe,—mainly upon the gravity (i. e., the mass) of the earth. There is no God and no law which dictates the course of action, but the things act on account of the inherent qualities which constitute them. The world is not a world of slaves, but a free play of uniformities. There is not a metaphysical or theological power that forces things, be they animate or inanimate, to pursue a certain course, but all things act in a definite and determinable way by virtue of their own nature. A thief steals when the occasion arises, and an honest man pursues the straight path of righteousness, as the cat will jump at the mouse and the oxygen will combine with the carbon. None of the events of the world happen at the dictates of either a God or a law, but because things are such as to act thus. Things consist of motor impulses, shaped by previous conditions, and, according to present conditions, taking a definite course.

Is this not atheism? May be it is. All depends upon the definition of the word. We must not be afraid of words; and if we find that atheism is right, let us frankly confess that we are atheists. Thank God that the days are past when atheists were burned at the stake, and let us be assured that, on the one hand, the best theist is more of an atheist than he may grant you; while, on the other hand, what is more important in our present discussion, the most rabid atheist is more of a theist than he himself is aware of. Let us see how. We shall start on the much abused road of the atheist and grant all that can be granted him.

A scientific world-conception needs no God. Laplace answered Napoleon, when asked why there was no mention of God in his *Celestial Mechanics*, "I have no need of that hypothesis," and every man of science can give the same answer, in the same sense. Further, in ethics, there is no need of the God-idea to teach morality. The God-idea is a convenient assistance to the teacher, but a moralist of a solid philosophical education is not in need of God. Kant, for instance, opens his work on *Religion Within the Limits of Pure Reason*, as follows:

"Morality, in so far as it is based upon the conception of man as a free being who binds himself through his reason by absolute laws, is in need neither of a superior being than himself to recognise his duty, nor of any special motive than the law itself that is to be observed."¹

Nor can it be said that our heart is in need of God. Christian mystics constantly have the word God in their mouth, but closely considered, the God of Jacob Böhme, of Johannes Tauler, and other theosophical philosophers is not very different from the Buddhist Nirvâna, and we might as well express the very same sentiments in an atheistic terminology.

We may grant even more. The craving for prayer which appears to be ingrained in the human heart seems to demand the existence of a God; but what did the Son of Man, who was conscious of his Sonship of God, say when his disciples requested him "Lord, teach us to pray!" He taught them a prayer which may be char-

¹ Preface to the edition of the year 1793. Leipzig: Modes and Baumann, Vol. VI., p. 161.

acterised as a prayer of weaning. The Lord's Prayer is a prayer only in its form ; in its substance it is a vow to abandon prayer in the literal sense of the word. If a friend of yours prays you to do him a favor, he attempts to induce you to comply with his wish. The Lord's Prayer attunes the heart to comply with God's ordinances, whatever they be, and to submit to his will. "Thy will be done" is not a prayer in the proper signification of the word. There is but one prayer in the Lord's Prayer which appears to be a genuine prayer, and we have reason to believe that it suffered by being translated into Greek, viz.: "Give us this day our daily bread." The oldest versions vary greatly, and a reconstruction in the original Aramaic which is now being attempted will at best be hypothetical. Judging from other passages which express the views of Jesus of Nazareth on the same topic, the significance of the fourth prayer will be clear, if viewed in the light of the sentence, "Take no heed of the morrow." Accordingly we are inclined to interpret it in the sense: "Let us be satisfied with our daily bread," and thus the fourth prayer would, like the other prayers, be a mere variation of the general theme expressed by Christ in Gethsemane: "Not my, but thy, will be done."

Here we have a remarkable coincidence between theism and atheism. Buddhism, commonly regarded as an atheistic religion, rejects prayer as an irreligious practice and replaces prayers by vows. Analyse the Lord's Prayer, and it consists of self-exhortations, of vows, which serve the educational purpose of a high-minded self-discipline.

* * *

Having made all these concessions to atheism, we shall now build our God-conception upon the very foundations which atheism leaves us. There are uniformities of a definite type in nature, which render it possible to describe natural phenomena, and even predict the course of events to come. These uniformities are the conditions of science. On the one hand they make the phenomena of the world classifiable and thereby comprehensible; and on the other hand they make possible the development of an organ of comprehension called "reason." Reason is simply the faculty of

tracing samenesses by designating the same type of phenomena by the same name. By comprehending samenesses we can anticipate the future and by anticipating the future we learn to seek the useful and to avoid evil. Thus, uniformities naturally produce purposive action. The apprehension of future results leads to adaptation, and adaptation pursued with conscious intention is the condition of ethics.

The uniformities of nature constitute in their totality a grand harmony which is commonly called the cosmic order; and this cosmic order comprises the motions of stars no less than the chemical combination of atoms, and is the principle which permeates the realm of man's life, including his highest intellectual and moral aspirations.

What is the *raison d'être* of these uniformities? Have they, such as they are, been ordained by the Creator, or are they accidental similarities? Here lies the whole God-problem in a nutshell, and this is the answer that science gives: "They are neither ordained, nor are they accidental: they are intrinsically necessary."

We can best explain the peculiar meaning of the term "necessary" by a reference to mathematics and logic. The philosophical term "necessity" must not be confounded with "compulsion." Philosophical necessity, in the sense in which we use the term, does not imply the curtailment of liberty, but denotes simply that certain things (including the future course of events) are conditioned in a definite way; they can be described; their nature, their behavior, their fate, can be determined by descriptive formulas. If a wrathful man is not checked by the fear of punishment, he will, if he be unimpeded, maltreat or even slay his adversary. He commits the crime of his own free will according to the character of the motor ideas of which his soul consists; he acts as he wants to act, without any external compulsion and yet with necessity. It is that inner necessity which is determined by himself, by his own character. In the same way the needle of the compass points toward the north by virtue of its magnetic nature. According to the physicist's interpretation of the process, the needle, when left at liberty

to adjust its position, will adapt itself to the magnetic lines of force that pass through it.

Mathematics teach us to comprehend the nature of necessity, in the philosophical sense of the term. "Necessary" is not that which suffers violence or is forced by some external pressure, but that which is definitely determinable. $1+1=2$ is a statement which carries with it an intrinsic necessity. The same is true of $2+2=1+3$ and of all arithmetical and geometrical theorems. Thus the sum of the angles of a triangle in Euclidean or plane space is always equal to two right angles; and all angles having their vertices on the circumference of a circle and subtended by the same chord are equal. There is no compulsion whatever here, but there is necessity,—that which in common parlance we call "a matter of course."

Mathematics, logic, and all other formal sciences are purely ideal constructions. An action is done, or rather, it is thought, and the results that are thereby established are noted. The results will always be the same if the process of construction be the same, and thus an ideal—i. e., a purely mental—world of samenesses, of uniformities, is established, which when applied to the realities of the material world serves to classify its phenomena, to describe them, and to predict their future course. The formal sciences in brief describe that which is necessary and arrange all necessities into a methodical and comprehensive system which assists us in seeing at a glance that, given some function under definite conditions, certain results will take place as a matter of course. Here lies the explanation of the cosmic order with all that it implies, science, purpose, ideals, and ethics.

Now mathematics and all the other formal sciences are descriptions, they are a system of formulas, and the question arises, Are these formulas pure inventions, or is there any reality that corresponds to them? There are philosophers who claim that the formal sciences do not formulate truths but are a *lusus intellectus*, a mere play of the mind. Even Kant took the word "ideal" in the sense of "subjective" which practically changed all ideal conceptions into imaginary magnitudes. On this little mistake, viz., the

identification of "ideal" and "subjective," which is inadvertently made by the great Königsberg thinker, hinges the philosophy of his critical idealism with the bold assumption of the ideality, that is to say the illusory nature, of space, time, and the categories. No wonder that his system lacked system and opened a loophole for agnosticism which has appeared in the shape of the doctrine of the unknowableness of things-in-themselves. Thus he arrived at a conclusion in which his radicalism offered a safe refuge to the reactionary obscurantism of his time, and Kant himself made the proposition that he "must abolish knowledge to make room for belief," which stands in an irreconcilable contradiction to his original aspiration for certainty of knowledge by avoiding both dogmatism and scepticism, i. e., Wolf's dogmatism with its unfounded assumptions, and Hume's scepticism, which is a denial of strict science, changing labor into sport, certainty into opinion, and philosophy into philodoxy. Kant aspired after certainty and arrived at nescience.

Let us briefly characterise the nature of the term "ideal," in the sense in which Kant ought to have used it when speaking of Space and Time as being ideal, as belonging to the realm of ideas!

An idea is a mental picture representing some objective reality. The objective reality need not be a concrete thing, but may be a general quality or a universal relation; it may be a combination of things not yet realised, and it may be a mode of conceiving mere relations under a common aspect. In all events it must be representative, it must point beyond or outside itself, it must be a symbol of something. The nature of ideas is their significance, that is to say, ideas are not purely subjective; they are subjective pictures of objective presences of some kind. Thus the ideal (things belonging to the realm of ideas) is not *eo ipso* identical with the purely subjective or illusory, but on the contrary, its most characteristic feature is representativeness, as signifying some objective reality.

Now we ask, What is the nature of the realities represented in the purely formal sciences? They are, John Stuart Mill and the whole Empiricist School notwithstanding, real presences in the

world. We may call them the purely formal laws of nature if we bear in mind that they are not laws, not coercive factors, but intrinsically necessary uniformities. Nor are they many various uniformities; they form one great system. They are one throughout and every special formula is but one aspect under special conditions of the same fundamental idea which may be comprised under the name of "universal consistency." They form in their totality an organic whole, a body of necessities which are all equally self-evident and even in their greatest complexity a matter of course. And they are objective presences in the real world of material things. They are the formative factors of the world. While all things are transient, they are eternal; while all existences are subject to change, they are the same forever and aye. They are uncreated and uncreatable. They are the measure of truth and standard of right and wrong. In a word, they are glimpses into the realms of the eternal. But consider! Geometry, arithmetic, and logic are only partial glimpses into the glorious harmony of the divine constitution of existence. There is also, as it were, a mathematics of ethics, and a geometry of religious aspirations, the practical importance of which is more easily felt than understood.

From the eternal moulds of these formative presences of existence all things proceed, and in them the forms of all things are preserved in a universal and superreal existence which knoweth not of origin or dissolution, not of birth or death, not of the anxieties of life and the fear of annihilation.

The plural form of the realities which correspond to the ideas of the purely formal sciences, is justified only because we become gradually acquainted with them. They appear to the growing intellect as a plurality of factors. But the truth soon dawns on a thinking mind that they form one grand system. We spoke of them purposely in the plural, for the sake of not anticipating the main implication of the God-idea, which consists in the organic unity of the world-order, as one consistent body of uniformities which may, not inappropriately, be spoken of as a personality, not human but divine, not bodily but spiritual and ideal, not individual, i. e., in a special place and having states of temporal succession,

but eternal and omnipresent. It is obvious that the unity of all formative factors is their most essential feature, for every single aspect is complete only when viewed as an aspect of this whole organism. When we try to realise the importance of these presences as a unity we shall soon find that they possess a direct and personal relation to the life of every one of us, which (if we remain conscious of the allegory) may very well be compared to a father, a lord, a judge. Besides it partakes of all those qualities which have, since time immemorial, been regarded as the characteristic features of the Deity. In the unity of these presences we have the Allhood of existence, which is the formative principle of the world, constituting the cosmic order. This Allhood is omnipresent and eternal. It comprises everything in its loving embrace. Man has originated in its image, and reason is but the reflexion of its intrinsic consistency. It is the prototype of logic, or, as the Fourth Gospel has it, "The Logos that was in the beginning," and thus it is the prototype of all truth. It is the world-reason, or, as it is called by the Taoists, the Tao of which Lao-Tze the venerated founder of Taoism says:

"It quickens all things and cherishes them;
It quickens but owns not;
It acts but claims not;
It brings up but rules not."

This Allhood is not an imaginary assumption, but it is the most real factor of life. It is not only real; it is superreal in the literal sense of the word, that is to say: It is not only a presence in the concrete things of this actual world in which we live; but it applies generally and would be no less present in any possible world that might originate somewhere, somehow, as if by magic. Nay, it holds good for purely fictitious worlds which, after the fashion of fairy tales, endeavor to establish other laws and arrangements, that would supersede the laws of nature with which we are familiar. Even the dreams of magic do not abolish causation; they only attempt to alter its concatenations, and its miracles are viewed as necessary results from the supposition on which they are based.

So little can we, even when we fly into the realms of fancy, discard necessity itself.

The attempt has been made to explain the world-order as a creation of God, but here the traditional conception of theism breaks down. For this omnipresence that permeates all existence is beginningless and intrinsically necessary. If this formative omnipresence be not God, if it is assumed to be different from God, then it is undoubtedly God's superior. God would have to comply with it, in order to construct the world correctly; he would have to utilise its norms of logic and consistency, in order to be wise; he would have to adopt its eternal principles of truth and right, in order to be moral. In other words, it would be the ultimate authority of God himself. It would be the deity to which even the creator was subject.

A duality of a Divine Omnipresence and a God-being is obviously absurd. For a God-being that is subject to the eternal norm of rationality, of truth and righteousness, is not God in the sense that we should use the term to-day. Such a God would be a being, a creature, an individual existence; it would be a god, but not God; it might be a Jupiter or a Brahma, or a world-soul. It would be much greater than any human being, but would still remain a creature such as we are, not the Allhood of existence, but a concrete, although ethereally spiritualised entity.

We need not enter into all the difficulties into which the assumption of a distinction between God and the omnipresent world-order would lead us. At any rate: a God-conception which individualises God and conceives of Him as a concrete being is mere paganism, whether or not it assumes the name of Christianity.

But while we are conscious of the symbolic nature of the various terms of the God-idea, we must not look upon them, when considering their significance, as untruths. The main difficulty of a purified God-idea probably consists in understanding that truths, as well as all things that are purely formal and unmaterial, are after all undeniable presences, possessed of real effectiveness in this world of actualities.

Let us see whether by purifying the traditional God-conception, we have lost anything of its religious significance.

All the attributes which have ever rightly been predicated of God are here combined in the Divinity that shapes the ends of the cosmos as a whole as well as in all its details, where we are confronted with immutability, omnipresence, eternality, universality, omnipotence, omniscience, justice, omnibeneficence, and an all-embracing love, long-suffering, and mercy.

Every one can readily see that the God of philosophy is immutable, eternal, universal, and omnipresent, for God is defined as the abiding in the transient, as the law of uniformities in the variety of natural phenomena. Although it may be difficult to realise vividly in one's mind God's omnipresence, from which no one can hide even his most secret thoughts, the idea itself is clear enough. But a few words are needed on omniscience, omnipotence, omnibeneficence, long-suffering, and love.¹

It is understood that the omnipresence of the formative factors of the world is not possessed of a knowledge such as is man's. It is a higher kind of knowledge; it is omniscience, not science. Science is discursive, walking as it were on crutches and proceeding step by step. Omniscience is argument and conclusion in one. It can dispense with investigation because it possesses the result before it searches for it. It is the automatic workings of the truth which appears in the unflinching correctness of so-called natural laws.

When speaking of God's omniscience we are apt to think of his thoughts as being like ours, transient and discursive, but they are eternal and omnipresent, and in this respect infinitely different from human thoughts. Whenever we are confronted with a truth that is found to be eternal and intrinsically necessary, be it a norm of reason or a law of nature, we are in the presence of a thought of God. Science formulates these laws, and every progress of science affords us a deeper insight into the character of God.

¹ I may be pardoned here for repeating myself. The next four paragraphs on the qualities of God appeared with slight additions and alterations in an article on "The Unmateriality of the Soul and God" in *The Monist*, Vol. VIII., No. 3.

Further, God's omnipotence is not a force that can be measured in footpounds. His strength is not power of muscle nor the might of armies. God's omnipotence is the irresistibility of His omnipresent will. It is the irrefragability of what appears to the scientist as the silent workings of natural law; it is the inevitable efficacy of the still, small voice, which on account of its apparent passivity, its long-suffering and patience, makes on the superficial observer the impression of non-existence. But experience teaches that its quiet ways are unfailing.

Finally, as to God's omnibeneficence, it is natural that primitive people did not see the goodness of God. They were too dependent still on the forces of nature to see the deeper aspects of the divine law that works for progress in the intellectual world and not less in the moral world. The soul of the savages contains too little of God's true nature to know Him correctly; their faculty of perception is still too dull, and therefore they see Him only in the thunder-storm and hear Him not in the still, small voice. But for that reason God is in every one of us, cherishing all in loving embrace. The still, small voice is speaking, addressing every one of us personally, but we must learn to listen.

And what do we gain by a purification of the God-idea?

First of all, the idea of God has thus become not only scientifically tenable but an intrinsic part of all science and philosophy. It is recognised as a name comprising all that which is the bread of our spiritual life. This God-conception reconciles Religion and Philosophy and affords a basis for a truly scientific theology.

Secondly, it transfigures tradition; it fulfils its aspirations without destroying its ideals. It explains the purport of the symbols of religious truth and teaches us to distinguish between the essential and accidental. This God-conception reconciles Religion and History and explains the errors of the past as necessary phases in a normal development, which, according to the law of evolution, is increasingly tending truthward.

Thirdly, it will liberate us from the bondage of the letter. We shall thereby learn to distinguish between symbol and truth, ritual and its significance, mythology and religion, dogma and doctrine,

that is to say, the lesson of the dogma. The pious need no longer fear Bible criticism and the destruction of their cherished idols; for they will understand that the fundamental truths of religion are based upon the recognition of that which is eternal. This God-conception digs down to the bottom rock of the conditions of a religious world-conception and affords a foundation which can never be shaken in the progress of science and civilisation.

These are great advantages, which will be appreciated by all those who have ever seriously grappled with the problem of the existence and nature of God.

Before we close our discussion I wish to touch briefly on a subject which may to many yearning souls be of incalculable importance. I wish to forestall misinterpretations which will actually arise in those to whom the present view is yet novel.

This purer God-conception loses nothing of the definiteness and personality of the old God-conception. A surrender of the letter does not imply a surrender of the spirit that God is our Father, our Lord, our Judge, our Comforter, our Saviour, the prototype of the incarnated Christ-ideal, the Way, the Truth, and the Light.

This God-conception is not the old pantheism which identifies God and the All. God is in all things, but He is also beyond and above all things. Nor is He the sum-total of all things. God is the Allhood of existence, but in addition he is the condition of any possible existence. He is *in* nature and yet different from nature. He is *in* reality but different from all real things. He is the supernatural in nature and the superreal in real things. He is the formative of things material, himself immaterial.

This God-conception does not teach the impersonality, but the superpersonality of God. God is the condition of all personality. God's nature is not an indefinite omneity, for He is possessed of a very definite character constituting the significance of existence as a whole and laying down the purpose of all existence, as well as imparting a definite direction to all life aspirations which finds expression in the evolution of solar systems, of nations, of individuals.

God is further not an indifferent being to us. He has a personal and private relation to all His creatures, being nearer to every one of them than the beat of their hearts and the neural vibrations of their brains. He is in them and yet different from them and infinitely high above them. He is their life, their home whence they start, and the goal whither they travel.

God is not like us, but we are like him. He is the light of our life, He is the mariner's compass which guides us, and the anchor of hope on which we rely. Unless we feel his presence, we shall find no peace in the restlessness of the world. Unless we sanctify our lives by the purport which his existence imparts to all life, we can find no comfort in our afflictions. Unless we recognise that our soul is an actualisation of his eternal thoughts, we shall not learn to fight the right way in the struggle for existence. Unless we listen to the still, small voice that teaches us our duties, we shall not obtain that blissful assurance which the childship of God alone can afford.

EDITOR.

DISCUSSION.

The preceding article on "God," which was twice used as a lecture, elicited before the philosophical club of the University of Chicago a lively discussion, in which problems closely connected with the God-idea were presented. On another occasion before the philosophical club of the University of Ann Arbor, where no discussion took place, the lecturer was afterwards privately interrogated by several inquiring minds. And since these topics may be of interest to our readers, we shall reproduce such of the questions and answers here as contain the gist of our conversations:

Question 1. "Your formulation of the God-problem contains two sides; one part of it is strictly scientific, the other religious. How do you unite both?"

I have endeavored to present a plain statement of facts and have then given a religious appreciation of those factors which shape the world at large and determine our lives, "roughhew them as we may." These factors are eternal neces-

sities: that is to say, we can understand that they must be such as they are and cannot be otherwise. They are not a plurality of factors, but are one throughout. They are uncreated and uncreatable, and therefore not the ordinance of a deity. They form an inevitable omnipresence in which all things live and move and have their being. On the one hand they are not an individual being of concrete existence; they are not here nor there; but they are truly everywhere at once. They are not this nor that particular existence which says "I am," excluding any other "thou"; they are not anything particular; they are the universal in the particular. Yet, on the other hand, they are neither nonentities. Although they are not concrete entities, they are none the less real. Indeed, they are the most important feature of everything real. They would remain the same, even though all material reality were annihilated. In this sense they are superreal. If nature did not exist, they would remain true; in this sense they are supernatural. They constitute the possibility of mentality and of moral aspirations and thus they build a higher realm of spiritual life upon the purely physical domain of existence. In this sense they are superphysical. Being the purely formal features of existence, they are the prototype of reason and the foundation of everything intellectual, mental, spiritual.

Although universal, they are not indefinite; on the contrary, they are the determination of every definite suchness in the world.

Although not particular and not individual, they are not lacking in that which constitutes personality; they possess a specific character which is sufficiently pronounced to lay down for all its creatures certain ascertainable rules of conduct and a standard of moral goodness.

These are facts concerning which there can be no disagreement; and they possess a direct bearing on our lives. They are the realities in the experience of mankind which were formulated under the name of God, and on our attitude toward them our entire life depends—our world-conception, our ultimate motives of actions, our moral ideals, our comfort in the vicissitudes of fate, our destiny in general.

Should this realm of the most important realities remain neglected? Should the superreal, the supernatural, the superphysical be left unheeded because its truth is more subtle than the grossly real, the crudely natural, the merely physical, the material? Certainly not. The historian can watch the growth of an appreciation of these higher factors of life in the development of religion which instinctively discovers the most salient moral truths and expresses them in allegories and parables. Are the parables untrue because they must not be taken literally? No, and a thousand times no! Religion is not the product of priestcraft but is the natural outcome of a groping after the truth. Mythology is the dawn of religion, as alchemy and astrology are the beginning of chemistry and astronomy. There is a close analogy between the religious and scientific evolution of man; and let us bear in mind that evolution has its phases; it passes through several stages; and if we have succeeded in attaining to the solution of a great problem, it will prove to be only a starting-

point for new problems. Evolution is never closed. Life is growth and completion, and consummation would mean death. The religious life of mankind is no exception. There are still higher vistas of a deeper religious revelation in store for us, and they will justify the religious aspirations of former periods. They will come to fulfil, not to destroy. They will teach us the reality of the still, small voice in the human heart and afford us a key to the significance of the mythology of the savage and of the parable as it was crystallised in mediæval dogmatism.

Question 2. "Is not your God-idea a mere abstraction and therefore lacking in the vitality which is indispensable for a religious conception?"

This question rests upon the assumption, which is quite common among many people, that abstract ideas are empty, unmeaning, and unreal. This is an error. Abstract ideas are, if they are but true, as significant as their poetical personifications; there is only this difference between the two, that while abstract ideas are more definite, the people who are not trained in exact thinking are more impressed by poetical descriptions than by concise formulas.

To attempt giving a philosophical definition of God in a missionary sermon addressed to the Zulus, or in our midst, to a Salvationist meeting, would be as much out of place as trying to teach mathematics or explain the falling of stones by the Newtonian formula to a child of three or four years. But because abstractions are empty and unmeaning to the unschooled, they are of the greatest importance and full of significance to those who have acquired the habits of exact thought. He who speaks of abstractions as being empty, only proves that he is still in the period of mental infancy for which the milk of mythology is alone the proper food. He can not yet digest the meat of scientific accuracy.

Question 3. "Is not a certain anthropomorphism allowable in speaking of God?"

Anthropomorphism in speaking of God is not only allowable, but, according to circumstances, even indispensable, for it is the means and the sole means by which the untutored masses, the half-civilised races, and all the many adult children that we find everywhere, can be approached.

Anthropomorphism was a necessary phase in the religious evolution of mankind and will remain indispensable even to the scientific thinker for the purpose of artistic and emotional expressions. Only we must remain conscious of our anthropomorphism and must avoid drawing conclusions from terms which are purely allegorical.

For instance, God is not a father in the literal sense. Take the allegory in the literal sense, and the highpriest, Ananias, was justified in denouncing the very thought of it as blasphemy. On this ground Mohammed rejects the Christian doctrine of the sonship of Christ. But understand that it is an allegory, symbolising God's intimate relation to every one of us, and it will be difficult to find a more beautiful and more impressive simile.

Question 4. "Do you regard this view of God as compatible with the Christian conception of God?"

It is not only compatible with the Christian conception, it is the Christian conception itself, in its matured and purified form. Any one who holds the traditional conception of God will, as soon as his mind becomes scientifically trained, be confronted with problems as to the nature of God. In the face of the truth that the world order is not made but intrinsically necessary and eternal, he can no longer look upon God as an individual being who makes worlds as the watchmaker makes watches. If it is impossible that God ordained those uniformities which are commonly called natural laws, the question rises, "Is God subject to certain universal necessities, or if not, what is his relation to them?" The solution here offered which regards every law of the cosmos, everything eternal in nature, everything universal in our experiences, as a part and parcel of God himself, will appeal only to those who have been confronted with the problem. Those who know of science and philosophy from hearsay only will not be in need of any reconciliation between religion and science, and we must excuse them for regarding the very attempt at comprehending the significance of God as a waste of time and idle talk.

Question 5. "Your conception of God is quite simple and apparently acceptable to the theist and the atheist. But it takes away all mystery."

Well! The purpose of every scientific and philosophical investigation is to do away with a mystery of some kind. An unsolved problem mystifies us, but when it is solved the facts are clear, and we might repeat with Schopenhauer, "*Simplex veri sigillum.*"

Question 6. "Is not mystery God's very nature?"

Many people love the mysterious and are afraid of clear thought; but if God really represented the mysterious, i. e., the inexplicable, and atheism clear thought or the solution of problems, the duty of science would be to reduce the domain of God to the utmost and if possible to let him entirely disappear. But God, if he is God at all and not merely the stay of superstition, is the light of the world, not its darkness; he appears in the order of the world and not in the supposed reversions of the world-order, commonly called miracles; he is the principle that pervades science, that conditions reason, and enhances progress, not a personification of obscurantism, ignorance, and reaction.

Question 7. "Do you not explain too much? Do you not explain God away and leave intangible relations, pure form, and natural laws in his place?"

Is it possible to explain too much? Does a phenomenon which is understood disappear? The reality of God remains the same whether or not his nature be understood. But we have the advantage of avoiding the errors connected with a literal belief in the allegories under which God is comprehended by the uneducated masses of mankind.

Do you think that music ceases to be music if we understand that the objective reality outside of us are air-vibrations, the intervals of which possess definite mathematical proportions? The beauty of music remains the same whether or not we understand its nature. It is the same with fire, electricity, life, and all other processes of nature. Fire was formerly supposed to be a peculiar stuff; it was regarded as one of the imponderable substances and was called *phlogiston*. Since we understand that fire is a mode of motion and not an imponderable stuff, we know that the existence of phlogiston is a pure invention of the misguided imagination of former scientists, but fire is as real now as it ever was. Thus that ultimate *why* of existence which by one word we call God, remains as real to-day as it ever was of yore, only we know to-day better what it is.

Question 8. "Is not the term superpersonal a mere euphemism for impersonal? If God is not an individual, he cannot be a person, for every person is an individual."

Allow me the counter-question, What constitutes personality? I grant that every person is an individual which is possessed of particularity, being bodily always at a given moment in a special place. A person in this sense is here, not there, and in this sense God is not a person. But the personality of man consists in his being possessed of reason and pursuing rational purposes. What is reason but the recognition of the universal? If the universal takes abode in an individual, the individual changes into a person, or in other words, it acquires personality. God is the principle of personality itself; ~~He~~ is the condition that renders personality possible. In ~~His~~ image man is made.

Question 9. "Is not a person, an individual, endowed with reason, or intelligence, sentiency, and will?"

Yes! But sentiency is not a quality which is typical of personality; it is not a feature that belongs exclusively to man; it belongs in the same degree to the animal. That which constitutes the characteristic feature of personality is the intelligence of the universal in experience which is rendered possible through language. Every man is, in this sense, more or less an incarnation of the Logos. In him the notion has originated of that which remains the same in all changes. He sees things (as Spinoza has it) *sub specie aeterni*, under the aspect of the eternal.

Allow me here to call your attention to the close connexion between sentiency and matter. All sentiency is particular; it is always in a special place and time; it is always awareness of material objects, involving material existence. Sentiency originates through a contact of matter with matter. Matter has been defined as the sense-perceived, and sentiency is the matter-perceiving. As to God, taking God now as the absolute Deity which in the Christian dogmatology is called God the Father, viz., the formative factors of the world, or God the Creator, we should say that He is immaterial and is not a sentient being. God is called "holy," which means, separated from everything sensual and sensory. Yet God possesses a definite

character, and His character determines the nature of truth and falsehood, of right and wrong, of goodness and badness.

While God is not an individual, he is not devoid of personality. A system of truths, or rather of such norms determining the actual world as can be formulated in statements of fact, in laws or truths, may be called a spiritual body, an organism, or a personality; for it is that which constitutes the personality of a person. In this sense God is not a person but a personality. Further, God manifesting Himself in evolution and culminating in the God-man, God as the divinity of this actual world of ours, the second person of the Christian trinity, is even as a cosmic principle endowed with personality. He has a will, or to avoid even the semblance of anthropomorphic expressions, he is the determinedness of the process of evolution. The universe has a definite character which is that which makes for progress, the onward motion of mankind, the power that makes for righteousness; the bliss of goodness, the curse of sin; in a word, the verdict of consequence which inevitably follows every deed according to the law of causation. God has a will, and He pursues a definite design; only His will is not like the will of a man; His will is the eternal determinedness of events; His design is not the plan of a Prometheus (i. e., a fore-thinker), a meditating plodder, a deliberate worker and constructor; God's design is the immanent, eternal order of things and the unalterable, immutable necessity that naturally produces the obviously noticeable fitness of things.

After these comments the remark will not be misunderstood, that God is not an individual being but after all he possesses personality; yet his personality is different from the personality of man, which is the personality of an individual, and therefore I prefer to call God superpersonal.

Question 10. "Is God conscious of himself?"

The question should be, How does God become conscious of himself? Consciousness is a representation of oneself for the purpose of adapting one's conduct to the requirement of circumstances. God as the absolute unity of the formative factors of the world, the ultimate norm of all existence in its superreal eternity, is not in need of consciousness and could not, without gross anthropomorphism, be said to be conscious of himself. But God in his practical attitude as the manifestation of the Divine in the process of evolution necessarily becomes conscious in the ideal aspirations of mankind which are pursued with purposive efforts of a morally determined will. God awakens to consciousness in man, and here we are confronted with the deepest truth of Christianity. The self-consciousness of God is realised in the God-man, in Christ, in the man of good will, in him whom Buddhists call the Perfectly Enlightened One, and Taoists the Superior Sage, or Holy Man. Mohammedanism saw in the trinity doctrine a tritheism; it returned therefore to a rigid unitarianism which conceived God in his absolute aspect alone. To the strict unitarian the word of St. John, "I and the father are one," is as much blasphemy, as the idea of God as being love itself, appears as a veiled atheism. Mohammed

declares that God is unbegotten and he does not beget. Accordingly he abhors the idea of Christ's sonship. From our standpoint the Christian trinity doctrine is a decided progress upon the prior unitarianism as it deepens the God-idea and dwells on the importance of the divine immanence in the evolution of moral ideals.

Question 11. "Do you believe that mere relations are real?"

I do not believe, I know. Belief of any kind is excluded from this scientific formulation of the God-idea. There is nothing hypothetical about it; it is a systematised formulation of facts. These so-called mere relations, or these purely formal uniformities, are not substances, are not energies, are not metaphysical essences of any kind; yet they are the determinants of the world.

Real means that which produces effects. The German *wirklich* shows the significance of the term in its etymology. *Wirklich* is that which *works*, or produces effects, that which determines the suchness of causation. Now, the purely formal uniformities are the determinative element of the forms of reality. If anything is real, they are real. Their reality is different from the reality of a stone or any other concrete object; but it is rather more than less real. The reality of a definite piece of matter is in one place, but the reality of the law of gravitation is ubiquitous, and this is not a matter of belief, it is a scientific truth, demonstrable in experience and verifiable by experiments.

Question 12. "Is not belief an essential element in religion?"

No! Belief is not essential, but faith is. Belief is imperfect knowledge. Should not the perfect be better than the imperfect? Belief is only essential to religion if the word is used in the sense of faith.

We must distinguish between faith and belief. The Greek word *πίστις* means faith, confidence, trust; and the Hebrew *אמונה* means firmness, reliability, trustworthiness. Belief in the sense of accepting unverified and unverifiable statements without investigation is not only not essential in religion, but downright irreligious. It is a sacred duty to inquire and gain as much light as possible on the main problems of life. To take for granted certain doctrines which are handed down to us by tradition, is immoral and must lead to the sanctification of superstition. What we need in life is not belief but faith. Belief is a matter of intelligence, or rather of neglect of intelligence; faith is a moral attitude. We need faith, i.e., faithfulness, firmness, stability, moral earnestness in life. We need the inspiration of good will toward all, above all we need a trust in truth. Indeed, we might condense the definition of the religious attitude to these three little words: religion is "trust in truth."

Question 13. "Do you think it possible that a clergyman could hold your views and remain in his church?"

Certainly I do think so. In fact, I have received letters from various clergymen—personally unknown to me—who thanked me for the light they had received from writings of mine on the religious question. Two of them wrote that, having

been affected by the contradictions between the letter of the dogma and the results of science, they had thought of leaving the Church, but now they saw their religious traditions in a new light and had thereby been enabled to find a conciliation between religion and science. They could now stay in the Church. They felt no longer the bitter self-reproach of hypocrisy, but could attend to their duties with a clear conscience and in gladness, as they had done in younger years before the scruples of doubt had attacked their souls. One clergyman told me that he had formerly seen no other way out of the difficulty than by turning to agnosticism for comfort, but now he saw that the religious evolution tended toward a scientific religion which would no longer be in need of mysticism.

How compatible a philosophical conception of religion is or can be with the active duties of a clergymen may be learned from the fact that Bishop Berkeley Herder, and Schleiermacher were bold and radical thinkers. I may also mention Pope Sylvester II. and other men of scientific distinction among the clergy. If my philosophy be incompatible with the position of a clergyman, the philosophies of these men, too, ought to be condemned as heretical.

I propose a philosophical interpretation of certain facts which have produced religion, the Churches, and dogmas; or rather I formulate the facts and show them under a certain aspect, that is all. I do not deny the facts; I deny only some unscientific explanations of these facts and replace them by a simpler explanation which abandons the antiquated metaphysical views and reconstructs the experiences of our religious life upon the basis of a rigid positivism.

Question 14. "But are not clergymen pledged to accept a belief in the letter of their dogmas?"

The pledges of clergymen when they are ordained are different in different churches. In some denominations they are very loose and allow much liberty; in others they are more direct; but, so far as I know, a belief in the letter is nowhere exacted. On the contrary, the most dogmatic churches give their members the greatest freedom of interpretation.

The faith of the church is laid down in the symbolical books, from the Apostolic Creed down to the most recent confessions of faith and platforms, but the very name "symbolical" implies that they contain truths which are not stated with scientific precision but in terms of symbols. The very first sentence of the Apostolic Creed, "I believe in God the Father," is an obvious allegory, and the allegorical nature of the term has never been denied. These formulations of doctrine are first of all historical documents; they must be treated with reverence and respect. We accept them as we accept the testaments of our parents and grandparents. They have been made to assist us, not to enslave us. It is our duty to think the same problems over again and revise the old statements in the fuller light of modern science.

P. C.

LITERARY CORRESPONDENCE.

FRANCE.

M. ALFRED FOUILLÉE'S *Psychologie du peuple français* is a clear, interesting, and finely conceived production. The main thing in such a task is not to discover the marked and distinguishing traits of a nation's psychology; what is needed is to set these traits into full relief, and to reveal their recondite connexions, for that portrait is most faithful in which every one recognises the old familiar features. But how have the characteristic traits of our nation been formed, to what sources are they to be referred—to race, to climate, or to history? In his criticism of these initial data, M. Fouillée has exhibited all the qualities of acumen and discernment which distinguish him. He finds himself ultimately led to the question of racial degeneracy, which at present is noticeably marked in the excessive diminution of French natality, and to casting a prognostic for the future, based upon his study of these facts as a whole. It is a delicate task, and one which touches at the same time the gravest problems of sociology.

In his introduction, where he treats of the factors which make up the national characteristics, as also in his first book on the European races and the part they represent in the French character, M. Fouillée submits to judicious analysis the current theories of race, environment, collective determinism, selection, etc. He points out their untrustworthiness in many things, their numerous breaches of logic, and ends by discovering in Gaul an aggregate of peoples which are distinctly characterised—Celtic, German, and Iberian (or, rather, Mediterranean) peoples. Gaul is not of Latin

blood; it is merely of Latin education. The Celt furnished to France a good initial material, solid and stalwart, but which was in need of amalgamation with a more personal, a more imperious, race,—an element which was afforded to it by the successive invasions of the Kymris, Visigoths, the Franks, and finally the Normans. The Mediterranean race added its contribution of flexible and lively intelligence, and also of tenacity. The fusion of the three races, writes M. Fouillée, ought to have ended by forming in France a rare and priceless harmony, “a sort of perfect accord, in which the Celt gave the tonic note, the Mediterranean the mediant note, and the German the dominant note.”

M. Fouillée, as we see, attributes to the Germanic element an important rôle—a rôle so fundamental and essential that he does not hesitate to consider the exhaustion of this element in France as the prime cause of the present weakening and impotence, temporary though it may be. He assigns, therefore, with Gobineau, Lapouge, and quite recently, Mismar, considerable value to anthropological sociology, or the doctrine which explains history by the qualities of races, although he avoids the extravagances of this method. He does not go so far as to forejudge the whole future of nations by the mere testimony of the color of the eyes and of cephalic indices. He does not omit to attribute a large share to acquired characters, to education, to the incessant action of events; in short, to the transformation of the original elements, whether by racial mixtures, or by the training and discipline of centuries.

In truth, it is no difficult matter to ridicule deductions drawn from certain physical characters in national types. Various degrees of brachycephaly or dolichocephaly are merely the gross evidence of certain qualities the mystery of which is resident in the cerebral substance. But these characteristic qualities of races are real. The measure in which a race remains plastic, and the degree to which its primitive outward characteristics contribute to determining its evolution in time, is another matter. We do not know what conditions are required for the formation of a new type capable of a full and happy development; this is a question of the future. It is a question which is now directly put to our country.

M. Fouillée discusses it; he does not slur over the unfavorable facts, and yet he arrives at conclusions which border on optimism, and which it would be out of place in me to call into doubt.

M. AUG. DIETRICH has translated from the Spanish the work of a writer who is very highly thought of in his own country, M. EDOUARD SANZ Y ESCARTIN. The title of the book is *L'Individu et la réforme sociale*; it does not proclaim a personal theory; it is made up of discursive, interesting and often eloquent studies upon the individual and society, labor, wealth, savings, property, capital, the social duties of the ruling classes, the attitude of the working classes toward the problem of bettering their condition, science, education, morals, religion, art, law, and the influence of woman. M. Sanz y Escartin is a high-minded and exceedingly liberal man; but he does not seek for social reform in a radical change of present conditions; he rather expects it from intelligent and consciously directed action, and for this reason shows no sympathy for the decadent scepticism of Renan, or for the mechanical doctrines of the socialistic systems, according to which every question is based upon political economy, and never upon the morality of men.

M. GEORGES RENARD, of Lausanne, presents us in his book, *Le régime socialiste* (the diametrical contrast of the preceding work) an idyllic picture of a society which has been constructed conformably to his own views. I shall not reproach him with having drawn up his outline-sketch with the utmost complaisance to his fellows. It is a requirement of our nature always to aspire after better conditions; the hard experience of life encourages such aspirations, and nothing is so unhealthy as moral indifference in the absence of all ideals. One cannot deny to the socialistic party the merit of having a belief, at least when it is represented by men of such kindly disposition as M. Renard. Socialisation, or the appropriation to common use of so-called natural properties (lands, mines, etc.), the suppression of inheritance, the valuation of individual production by the hours of labor (calculated by a very ingenious rule)—such are always the salient features of the doctrine. M. Renard does not conceal the difficulties attending the enforcement of

such a *régime*, and I doubt whether such a system would even give the good results which he hopes from it. Furthermore, what assurance have we, if this variation is introduced into the economical system, that social relations will vary in the precise direction which we desire and imagine they will vary? It is true that evolution is never ended, and that our societies will continue to change in the direction of some such ideal of justice as the socialistic school invokes, but it will certainly not be by the means which that school proposes. At any rate, if we survey the rich variety of social types which exist in the world, we shall be led to the conclusion that they can be reconciled only by the creation of a mixed type, and that there will be incorporated in the practical *régime* that portion only of the doctrines of the schools which is involved in the natural play of the moral and economical facts. Think only of the numerous changes that are being effected in the social sphere before our very eyes by the extension of trade, by the inventions of science, and by the potent intervention of the sentiments of modern man! On this last point I shall remark, in opposition to blind conservatives, that the profound emotions of human nature can be satisfied in different directions. Every religious or social organisation exhibits the state of the soul that corresponds to it, and it is sufficient to point in this respect, for example, to the Chinese type and the American type to grasp vividly the difference of the sentiments which actuate each of these peoples.

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The *Année sociologique*, under the editorship of M. EMILE DURKHEIM, is a companion volume to the *Année psychologique*, which M. BINET edits.¹

This publication contains: (1) Original memoirs, the first of which for the present year is by M. Durkheim and is entitled "La prohibition de l'inceste et ses origines," the second by Simmel, of Berlin, and entitled "Comment les formes sociales se maintiennent"; (2) Synopses of all the works and magazine articles of France and foreign countries which are of interest to sociologists,

¹ L'Année psychologique is now published by Reinwald.

—synopses which are grouped under special titles, so as to constitute by their arrangement a coherent survey of the subject. M. Durkheim, in his memoir, attributes the prohibition of incest to exogamy; that is to say, the prohibition of marriage between members of the same clan; and he at the same time points to the characteristic mark of the clan in the *totem*. All prohibitions of this kind which have been observed among lower societies owe their existence, according to him, to exogamy. But what are the determining causes? He rejects, for good reasons, the theories which attribute it either to some particular reason, as to the custom of infanticide of daughters (MacLennan), or to the inclination of primitive societies for war and pillage, or to some feeling for the bad results which are imputable to consanguineous marriages (Morgan). Resting his arguments upon the fact that the totem is a god, and totemism a cult, he is rather inclined to see in this custom a particular case of a very general religious institution, viz., the *taboo*. If now it be asked why the taboo applies to women, it would appear certain that the rigorous prohibitions in this regard must have been intimately connected with the ideas which primitive man held of menstruation and menstrual blood. And if finally we seek the reason why sexual interdictions applied exclusively to the members of the same clan, we shall find the reason in the fact that originally filiation was exclusively uterine, the children received the totem from the mother, and it was women through whom the blood was propagated which as a common possession constituted the unity of the group. To-day, doubtless, the prohibition of incest involves moral ideas which appear foreign to the practice of exogamy as thus understood. Nevertheless, M. Durkheim believes that the new habits have been actually engrafted upon the ancient rites, and his investigation shows that our present moral conception is the outcome of moral rules, in the formation of which it has played a ruling part; that it results, in fine, from elementary states of consciousness, variously combined, and masks the facts or customs which we could hardly have expected to find in its origin.

M. LE DANTEC, in his *Évolution individuelle et hérédité*, sets forth an application of his doctrine of life to the special and much-debated subject of heredity, notably the heredity of acquired characters, on which, it seems to me, he has been successful in throwing some light. He starts out by studying the conditions of existence of plastids, and he so gives more precision to the notions of life and death, of variation, of vital competition, etc., to which the biologists frequently assign a vague or arbitrary meaning, because they directly investigate the phenomena of life in the higher beings, which are already on too complex a scale. His theory is based, in fine, upon the consideration of the "chemical states" of elementary plastids, as also upon the conditions which determine their variations and their evolution, with the result that we are enabled to pass without a break from the consideration of monoplastids to that of metazoans.

"The chemical nature of the plastic substances of descendants is identical with that of the plastic substances of ancestors." Such is the simple principle by which M. Le Dantec defines species. "Species" for him embraces all varieties of plastids which differ only as regards the ratio of the respective quantities of their plastic substances without there being among them any qualitative difference. "Variation" is necessarily effected upon the quantity of the said substances, or upon their quality; and it persists just as long as the plastid which has varied remains in the same condition of chemical equilibrium. If we consider now not the isolated plastids but the polyplastidular beings which possess an "interior milieu," and subsist by correlation and by co-ordination of their component parts through a "skeleton," we shall discover that the effect of variation upon general correlation in the interior milieu is not manifested in the general co-ordination of the organism as it might have been before the fixation of that co-ordination by the skeleton."

The formation of the solid or rigid parts of the organism limits the field of living activity. This remark paves the way for a reconciliation of the theory of the Neo-Lamarckians, according to which acquired variations can be inherited, with the theory of the Neo-Darwinians, according to which they are not inherited; it is im-

portant to note, for example, that "specifically acquired" characters may remain "morphologically latent"; it is chemical character that is hereditary; the son will have, not the shape of the father, but the shape which the father would have had if his skeleton had not opposed his assuming the characteristic form of equilibrium of his chemical composition.

The adherents of the theory of germ-plasm (Weismannism) regard mortality or immortality as properties of the more or less differentiated histological elements, even when these are qualities of polyplastidular agglomerations of the metazoan *individual* itself. Another thing is the *general death* of the individual, the destruction of the co-ordination which makes of it a being. Further, there is the *elementary death* of the histological elements, the chemical destruction of their plastic substances. The aged and skeleton-laden body of a metazoan is made up of young and vigorous elements which would ask for nothing but life for an indefinite period if the proper conditions were furnished them, and this it is that takes place in fecundation. By this important distinction, the objection regarding the transmission of artificial injuries is removed. "It is thus unreasonable to demand that the skeleton should be reproduced in the child with the exact *artificial* lesion occurring in the parent, to require that a leaden bullet lodged in some tissue should be inherited; the leaden bullet may have an hereditary influence upon the organism, but this hereditary influence will not reproduce the leaden bullet."

The foregoing synopsis gives scarcely more than an idea of the work of M. Le Dantec. I recommend it to my readers, both for its critical parts, which are very rich, and for its doctrinal parts, which are extremely novel.

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Social science—to judge from the works which are now publishing, and which are announced for the immediate future—has for some time been attracting a host of workers; I refer to social science as a whole, including also religious studies.¹ Pure psy-

¹ The plenitude of the work in this department has created a special series.

chology appears to be somewhat neglected, and at times leans toward general philosophical speculation, with which every one was so much discouraged, or it is entering on more intimate relations with the experimental work of the laboratory, and with biological studies; the craze for the marvellous, for hypnotism, etc., which only recently produced so many publications, has died away, although I should not say it is still extinct; and thus the saying of Charcot is verified, who, fatigued one day by the noise and bustle of the movement remarked, "It is a fad which will pass away."

In psychology, I have to point out two volumes only: one by M. M. Binet and Henri, *La fatigue intellectuelle* (Schleicher, publisher), an excellent production, as was to have been expected from these two authors; and another by M. L. Dugas, entitled *La timidité*, an ingenious and readable monograph.

Dr. James Sully's work, *Studies in Childhood*, has been translated into French by M. Monod.¹ The work, which is accompanied by a preface by M. Compayré, exhibits all the fine qualities of this distinguished psychologist; I shall make but one remark regarding children and drawing. My studies upon this subject have led me to believe that the child, like the adult, does not represent to himself things that he has seen,—I should even say, he sees things only under the form of a combined motor and visual scheme; and that the difference between a child and the adult, and even between artists, springs particularly from the accuracy and wealth of this scheme, or of what I have called our "images of interpretation," or "images of translation"; that if the child is satisfied with his crude drawings, the reason is that the unfinished figure drawn by his awkward hand is, nevertheless, an adequate symbol for him of the reality which he has in his head; that the same species of illusion exists also in the accomplished artist, whom we see satisfied with a figurative symbolism, more or less learned (in the cartoons of Puvis de Chavanne, for example); that the psychological pro-

Bibliothèque générale des sciences sociales (F. Alcan, publisher), in competition with the *Bibliothèque sociologique internationale* (Girard and Brière, publishers).

¹ Felix Alcan, publisher. The remaining works, where the publishers are not mentioned, are also published by Alcan.

cess, in a word, remains the same from the childhood to the maturity of art.

I have further to mention the following authors and works: M. Cherfils, *Essai de religion scientifique* (Fischbacher, publisher); M. de Laveille, *Un Lamennais inconnu* (Perrin, publisher); and also the eighth volume of the *Année philosophique*, which is still edited by M.M. Renouvier, Pillon, et Dauriac.

LUCIEN ARRÉAT.

PARIS.

DISCUSSIONS.

ON FACTS AND OPINIONS.

Mr. Paul Shorey calls *Plato's Logic*¹ "a perverse book in support of a fantastical theory . . . a tissue of fallacious reasoning wrought on the frame of an impossible method . . . a series of fallacies resting on misapprehensions of the fair meaning of the text and context of his author." This is very plain language and I could have no objection to it, had my critic stated the above as his personal subjective opinion. But in truly American fashion he insists on his views as "a fact" and calls his subjective condemnation a "statement of facts"—believes that he has proved it "by citation and indisputable fact." In view of this, it becomes the author's duty to warn the readers of *The Monist* that they should not rely on such "facts." A fact is what can be proved to the satisfaction at least of a majority of competent judges. Now let us consider the following true facts which are in contradiction with Mr. Shorey's statements:

1. Mr. J. Adam, known by his investigations on Plato, says in the *Mind* for July: "There can be no question that Mr. Lutoslawski has rendered a great service to Platonic scholarship. Of his industry, zeal, and enthusiasm, it would be impossible to speak too highly; and his acquaintance with the literature of his subject is probably unique. His book is one of the most suggestive and stimulating works on Plato which the present generation has seen."

2. Monsieur Henri Weil, one of the most eminent Greek scholars in France, writes in the *Journal des Savants* for February, after a discussion on a point of disagreement: "Notre dissentiment porte sur une question qui a son importance, 'mait qu'on peut résoudre autrement que M. Lutoslawski tout en reconnaissant la 'grande valeur d'un ouvrage, fruit d'un puissant effort de synthèse et qui marque 'une date dans les études platoniciennes."

3. Prof. Th. Gomperz, author of *Die Griechischen Denker*, acknowledged to be the most competent scholar of Plato in Austria, said in the meeting of April 20th of the Kaiserliche Akademie der Wissenschaften in Wien: "Lutoslawski hat 'keine neuen sprachlichen Untersuchungen angestellt, wohl aber die Resultate der 'vorhandenen, deren Urheber zumeist von einander nichts wussten, in so gut als 'vollständiger Weise mit einem staunenswerthen Anfecht an Mühe und Sorgfalt, 'und mit einer nicht zu überbietenden Literaturkenntnis zusammengefasst und

¹ See the review of Mr. Lutoslawski's work by Professor Shorey in *The Monist* for July, 1898 p. 621.

"mittelst einer eigenartigen, von ihm Stilometrie genannten Methode zu bearbeiten und zu verwerten sich bestrebt."

4. Prof. Felice Tocco, the most competent authority on Plato in Italy, says in No. 1 of *Atene e Roma*: "Quest opera importante, che per la ricca bibliografia vince tutte le altre, e per la novità di alcuni risultati da tutte si dilunga, merita senza dubbio alcuno un attento esame, principalmente da pa parte di chi non sia affatto d'accordo con l'autore. Il quale da parecchi anni attende a questo lavoro, e le opere di Platone ha con grande studio frugate e rifrugate. . . ."

5. Professor Susemihl, well known in Germany for these forty years by his publications on Plato, though he dissents from the author on many points, writes in the *Wochenschrift für klassische Philologie*, No. 26: "Er hat mit kolossalem Fleiss die ganze einschlagende Literatur durchmustert und mit gutem Urtheil der Kritik unterzogen . . . hoffentlich erscheint ja von demselben bald eine deutsche Bearbeitung und sie wird lebhaft Discussionen hervorrufen."

6. Prof. H. Struve, of Warsaw University, the most competent authority on Greek philosophy in Poland, writes in *Biblioteka Warszawska*:

"Nie wahamy sie twierdzic ze . . . We do not hesitate to affirm that
"dzielo to na dlugie czasy zajmie jedno . . . this work will occupy for a long time
"z pierwszorzednych miejsc w dziejach . . . one of the first places in the history of
"Platonizmu. . . . Platonic philosophy."

The above facts might easily be multiplied and they are here given only as samples of opinions of competent scholars on Plato's *Logic* in England, France, Germany, Italy, Austria, and Poland. These samples are taken at random from many dozens of reviews of Plato's *Logic*, and they are sufficient to prove that Mr. P. Shorey's opinions are not "facts"—because if they were, I could not have deceived six among the most competent critics in six different countries.

Besides, there is in Mr. P. Shorey's article one strange mistake for which no possible reason could be found in my work. He speaks of the "short time" I have taken to study Plato. Readers of Platonic literature know that my first book on Plato was published ten years ago and must therefore have been written earlier.

It is an inevitable consequence of the limitations of human nature that students of any subject cannot be brought to agree on *all* points, and no sober critic expects from an author who produces a new method of investigation and new results the irrefutable proof of all his opinions. I have repeatedly insisted throughout my work on the necessity of further investigations and I have nowhere claimed to have given a final solution of all difficulties implied in the problem of the interpretation of Plato. But Mr. Shorey misquotes me and misunderstands me on purpose, only to justify the French quotation with which he began his slaughter-business. I shall limit my answer to his objections to a single point only, to show his method. In the last four chapters of my work, pages 363–525, I have quoted over fifty passages to support my view that for Plato in his later stage "ideas exist in reason." Mr. Shorey selects one quotation at random which admits another interpretation and makes the reader believe that my interpretation is meant as a "translation" of Plato's text. My quotations of Plato's text are always given only as evidence on which I have based my views about Plato, and there is scarcely any attempt in the whole work to translate the given texts which I suppose are intelligible to my readers without help.

Similar in method are the other observations of Mr. Shorey. As he seems to like French, I may conclude: *les hommes que vous tuez se portent très bien.*

ŁOMŻA, POLAND.

W. LUTOSLAWSKI.

BOOK REVIEWS.

SOCIAL AND ETHICAL INTERPRETATIONS IN MENTAL DEVELOPMENT. A Study in Social Psychology. By *James Mark Baldwin*, Professor in Princeton University, Co-Editor of the "Psychological Review." New York: The Macmillan Co.; London: Macmillan & Co., Ltd. 1897. Pages, xiv+374. 8vo. Price, \$2.60.

The present volume is a continuation of the author's studies in genetic psychology, published under the title of *Mental Development in the Child and the Race*, but is otherwise quite independent. Its thesis is the extent to which the principles of the development of the individual mind apply also to the evolution of society, an inquiry which deals with two main questions,—what are the principles of organisation, growth, and conduct operative in the mental life of the individual, and what additional principles, if any, are exhibited by society in its forms of organisation, progress, and activity? As pointed out by the author in his Introduction, there are several methods by which the inquiry could be conducted,—the Historical or Anthropological method, the Sociological or Analytic method, and the Genetic method. Of these, the first and second examine into the progress of social development and compare the results arrived at with what is known as to the development of the individual mind. The Genetic method comprises two fields of inquiry, the psychological and the biological, of which the former deals with the phenomena of human mental activity, and the latter with the phenomena of animal life, organic and psychical. Professor Baldwin prefers the *Psychogenetic* method, which he explains as being an inquiry into "the psychological development of the human individual in the earlier stages of his growth for light upon his social nature, and also upon the social organisation in which he bears a part." His work is necessarily, therefore, based on a study of child-life, and he states that its main thought is the conception of the growth of the child's sense of personality. Valuable as is the Psychogenetic method, it is evident that it is not complete in itself, or rather that its conclusions require verification by comparison with those derived from the other methods. Undoubtedly it is "based upon observed facts and may be controlled by them," but children vary so much among themselves that it is not always safe to generalise from observation of the actions of a few. More-

over, it must not be forgotten that the civilised societies of modern times have attained to an adult growth, so that there cannot be an exact parallel between the phenomena of child-life and those of social life. Hence the value of the Anthropological and the Sociological methods by which the childhood of the race is discovered, enabling the stages of development of society itself to be compared with those of the individual. It is only proper to say that Professor Baldwin recognises that fact, and himself occasionally employs these methods or makes use of their conclusions.

Professor Baldwin's work, which, as seen from its title page, has been crowned with the gold medal of the Royal Academy of Denmark, is divided into two books which treat respectively of the Person and of Society. The subject of Book I. is dealt with under the four heads of The Imitative Person, The Inventive Person, The Person's Equipment, and The Person's Sanctions. Part I., in which the Imitative Person is considered, is intended to be a more or less complete study of social and ethical psychology, so far as its topics go, and its concluding section contains a summary of its argument which gives an excellent idea of the nature of the work as a whole. "All our thought," says the author, "has led us to see that one of the historical conceptions of man is, in its social aspects, mistaken. Man is not a person who stands up in his isolated majesty, meanness, passion, or humility, and sees, hits, worships, fights, or overcomes another man who does the opposite things to him, each preserving his isolated majesty, meanness, passion, humility, all the while, so that he can be considered a 'unit' for the corresponding process of social speculation. On the contrary, a man is a social outcome rather than a social unit. He is always, in his greatest part, also some one else. Social acts of his—that is, acts which may not prove anti-social—are his because they are society's first; otherwise he would not have learned them nor have had any tendency to do them. Everything that he learns is copied, reproduced, assimilated, from his fellows; and what all of them, including him,—all the fellows, the socii,—do and think, they do and think because they have each been through the same course of copying, reproducing, assimilating, that he has. When he acts quite privately, it is always with a boomerang in his hand; and every use he makes of his weapon leaves its indelible impression both upon the other and upon him!" After referring to the unsatisfactory state of social discussion, arising from the neglect of such facts, the author continues: "Once let it be our philosophical conviction, drawn from the more general results of psychology and anthropology, that man is not two, an ego and an alter, each of which is in active and chronic protest against a third great thing, society; once dispel this hideous un-fact, and with it the remedies found by the egoists, back all the way from the Spencers to the Hobbeses and the Comtes,—and I submit the main barrier to the successful understanding of society is removed."

It is somewhat strange to find Comte classed with the egoists, considering that his motto was "live for others," but apart from this the above paragraph is quoted

owing to its reference to the ego and the alter as opposed to society. In Professor Baldwin's system the growth of the child's personal consciousness has three stages, the *projective*, the *subjective*—that in which what was before projective becomes, through the exercise of the function of imitation, subjective—and the *ejective* in which the child refers to others the bodily experiences it is itself conscious of. The third stage witnesses the birth of the social self, the real self, which is *bipolar* as being constituted of ego and alter. These are born together, each being an imitative creation of the other, and each therefore being a *socius*. "The development of the child's personality," says the author, "could not go on at all without the constant modification of his sense of himself by suggestions from others. So he himself, at every stage, is really in part some one else, even in his own thought of himself. And then the attempt to get the alter stript from elements contributed directly from his present thought of himself, is equally futile. He thinks of the other, the alter, as his *socius*, just as he thinks of himself as the other's *socius*: and the only thing that remains more or less stable, throughout the whole growth, is the fact that there is a growing sense of self which includes both terms, the ego and the alter." This notion of self as a *socius* is excellent, and is consistent with the fact that each individual is *organically* a *socius*,—a unification of numerous elements which represent ancestral activities and give rise to what may be termed ancestral tendencies. If such be the case, then the imitation and the invention which form such important features of the child life should be regarded as efforts to reproduce what had been acquired by the past experiences of the general *socius*, or, from the individual standpoint, the personality which was the particular outcome of such experiences.

This consideration shows that the individual must not be merged entirely in the society of which he forms part. His emotions and sentiments, his instincts and intelligence, which form what the author terms "the person's equipment," although they are aroused into activity and guided in their action by the social environment belong to himself, as also do the "personal sanctions" which justify his conduct. Although the individuals belong to society, yet this is constituted of individuals who do not live that they may benefit society, although if they live rightly they will necessarily do so, but that their personalities may be fully developed in continuation of the growth of the sense of personality which takes place in the child through its social experiences. It is true, that without the aid of his social training a child left to itself would be little, if anything, above the animal. But this proves only that society is essential to the development of the personality, and it may be that, as language, play, and art are to be treated as social aids to invention in childhood, so society itself must be regarded as an aid to further personal development, and as having in the absence of this no actual *raison d'être*. Society undergoes development *pari passu* with the individual or with the series of individuals through whom it is perpetuated from generation to generation. Professor Baldwin, in his admirable treatment of the Genius as an inventor, points out that the new

thought leads to "a precipitation about a new nucleus" requiring new social adjustments.

The proper view as to the relation between the individual socius and the general socius is that they are complementary, each living for the other as well as for itself. This is the view which would seem to be required by Professor Baldwin's conclusion after drawing a parallel between the three stages he describes in the child's bashful period and the early ages of the race. The third stage is reflective, which is simply the way in which the child gets a notion of himself; and it marks the development of his personality, "wherein he has to give, by the very movement of his own growth, due value to the two terms which lead him on,—the ego and the alter." Reflexion, by which is meant turning round and examining something in consciousness, thus distinguishing the not-self from the self, is said by the author to be born "of the need of getting a sort of accommodation which will reconcile the personally aggressive or instinctive with the personally imitative or spontaneous," and the race proceeds in a similar way. It had to reconcile the instinctive tendencies derived from animal ancestry with the co-operative tendencies which social life required, and thus "the race became reflective, intelligent, and so started on a career of social development in which the two fundamental influences were to work together,—the private selfish interest and the public social interest." Professor Baldwin supposes that the social tide set in when man discovered that he could exist without killing and being killed and with the invention of the arts of tilling the soil and living, partly at least, on vegetable food. But it may be doubted whether anthropological inquiry justifies this opinion. It is found on the one hand, that a form of society exists among the lowest savages of the present day some of whom, such as the aborigines of Australia, do not till the soil; and, on the other hand, that tribes who do practise this art are almost constantly at war with one or other neighboring tribe. The islanders of the Pacific furnish examples of this case, as did at one time the native tribes of North America. Moreover, the peculiar social conditions which would seem to have subsisted almost universally with primitive man, precluded such a conflict between private selfish interest and public social interest such as Professor Baldwin supposes to have existed, at least to the extent which history would require. The real distinction was between the gens and the tribe, which represented the external interests of the community as the gens represented its internal interests. These are so well looked after that internal dissensions are far from being common. Selfishness in dealing with neighbors, however, was almost universal long after the agricultural and pastoral stages had become firmly established. At the same time there was probably an internal struggle going on between the gentile institution and the individuals who administered the affairs of the tribe, which resulted in most cases in the substitution of father right for mother right and the consolidation of all interests as a tribal organisation, with father right instead of mother right as the recognised mode of descent.

These facts are mentioned to show that it is not safe to theorise too far unless the actual facts as ascertained by anthropological research are fully considered.

Book II. of Professor Baldwin's work is divided into three parts, which treat of the Social Forces, Social Organisation—which is regarded as being due to a continuation of the two-fold exercise of the imitative function on which the growth of the individual's "self-thought" is dependent—and Social Progress. As to this we can say only that it is declared to be "necessarily in the direction of the realisation of ethical standards and rules of conduct." This work proper, after a chapter dealing with rules of conduct, concludes with a General Retrospect of the relations between the society and the individual. In a series of Appendices, the views of Professor Royce and other writers on kindred topics are considered in some detail, adding much to the value of a very important and highly suggestive work, on the completion of which Professor Baldwin may be congratulated.

C. STANILAND WAKE.

OUTLINES OF SOCIOLOGY. By Lester F. Ward. Author of *Dynamic Sociology*, *The Psychic Factors of Civilisation*, etc. New York: The Macmillan Company; London: Macmillan & Co., Ltd. 1898. Pages, xii, 301.

This volume is substantially a reprint of twelve lectures given by the author at the School of Sociology of the Hartford Society for Education Extension in 1895, and printed subsequently in the *American Journal of Sociology*. It is dedicated to Dr. Albion W. Small, "the first to draw attention to the educational value of my social philosophy, the staunch defender of my method in sociology, and to whom the prior appearance of these chapters is due." This dedication at once draws attention to what are the two most important points in connexion with any philosophic system—its principles and its practical value, and these points will engage our attention. The first part of Mr. Ward's work is devoted to the consideration of Social Philosophy, and the second part to that of Social Science, although the author states in the Preface, somewhat inconsistently, that "the earlier chapters may be regarded as aiming to show what sociology is not, while the later ones have for their object to set forth in broad outlines what sociology is." The latter will undoubtedly be the more interesting to the general reader, although the earlier chapters are by no means of a purely negative character, and may be considered as introductory to the treatment of the actual subject, and essential to the educational object the author has in view.

In dealing with the positive aspect of sociology, Mr. Ward very properly treats the "social forces" as natural forces, for which he would find authority in the teaching of Lord Bacon if such were needed, and it might be supposed, from the title he has given to the second part of his work, that he regarded sociology chiefly, as a matter of science. This would be a mistake, however, as *apart from its applications*, which constitute art, science is a mere "knowing" and not of much real use. Sociology is thus on its practical side the "social art," and the author, in ac-

cordance with this view, affirms that true legislation is *invention*. Actual legislation is rather prevention than invention, but it is declared to be faulty, as natural forces ought to be utilised instead of being checked and curbed. The aim of government, as "the art that results from the science of society through the legislative application of sociological principles," is the modification of the phenomena so as to serve man, which requires first the perception of the proper modes and then "the necessary adjustments to secure the useful end." The difficulty with this view is, that it regards society as a kind of Garden of Eden where the plants require only to be cultivated to bring forth good fruit, although otherwise they would run wild. It does not allow for the fact that there are weeds which require preventive treatment, the correspondents to which in human life are the hereditary criminals, those who have a tendency to do evil, not only through acquired habit, but also by defective structural organisation. Moreover, although such men and women may be regarded as survivals of, or rather from, an earlier stage of human progress, that of almost pure savagery, yet it cannot be denied that the average man only too often shows much the same spirit in his dealings with his fellows. Greed of some sort is the source of most of our social evils and a great proportion of legislation and judicial administration is directed towards preventing or remedying its action. Mr. Ward thinks that, notwithstanding the "inane floundering" of the American House of Representatives, which show how little reason has to do with democratic legislation, democracy has solved the moral side of the question of government, seeing that it *wishes well* for the people. What is wanted is increased intelligence, and the author's conclusion is that "if the social consciousness can be so far quickened as to awake to the full realisation of this truth in such vivid manner as to induce general action in the direction of devising means for the universal equalisation of intelligence, all other social problems will be put in the way of gradual but certain solution." This reads as though intelligence, like wealth, was accumulated in a comparatively few hands, and that if a kind of mental irrigation could be brought about by distribution of the accumulations all social evils would be remedied. That view does not speak well for the school system which is supposed to have made the Americans the most generally intelligent of peoples, and it is hardly consistent with the fact that, notwithstanding the spread of education, the moral outlook is by no means encouraging. To say nothing of the lack of moral fibre shown by the great number of divorces which take place, the increase in crime keeps pace with that of wealth rather than that of education, and we are compelled, therefore, to believe that something more than increased intelligence is required to ensure the solution of the existing social problems.

That Mr. Ward takes the right view as to the nature of intellect when he speaks of it as the directive agent, that which guides the feelings, cannot be denied. The intellect is the telic power which has given man the control of the vegetable and animal kingdoms and finally of the physical agencies at work in nature, and it is supposed to have been developed as "an aid to the will for the better satisfac-

tion of desire." Intelligence is the process of "converting means into ends," that is for obtaining what is desired. But what is to ensure that the desire shall deserve to be thus gratified? Morality is not wholly altruistic. Moreover, intelligence and reason are not the same, although they appear to be so regarded by Mr. Ward, who errs also in his view as to the origin of intellect and therefore of reason. When considering the question whether animals can reason, he speaks of the great intelligence of certain animals as favoring an affirmative answer, and in fact he places the "intellectual" attributes in opposition to the affective faculties, giving the rational faculty to the former. But what are called the beginnings of reason in the dog, elephant, and other animals are merely evidences of a high degree of intelligence. Reason, as thus distinguished, requires for its action the use of symbols such as are embodied in human speech, or in the more refined language of mathematics. This Mr. Ward speaks of as a purely human power—although he speculates on the possibility of some animals being able to say certain words and having an articulate language—and reasoning, properly so called, must be regarded as a purely human attribute. Reason has much the same relation to intellect as self-consciousness has to consciousness. The author quotes with approval Schopenhauer's statement that animals have consciousness but live without *Besonnenheit*, which he says "seems to touch the kernel of the problem." It is true that the German philosopher speaks, in the language of his time, of self-consciousness as an intellectual process, but it is evident that he regarded it as requiring thought, and this is not possible without the use of the symbols which are usually referred to as the instruments of reasoning and which are essential to it. The distinction between intellect and reason is really made by the author himself when he speaks of the "higher powers of introspection, speculation, reflexion, abstraction, and generalisation which characterise the developed mind of man." The undeveloped mind of man possesses these powers to some extent, but they are wanting to the animal mind, although it may exhibit great intellectual activity, even in the choice of means to ends. The practical application of the matter is that the solution of the social problems which, as we have seen, Mr. Ward believes will be attained through the "equalisation of intelligence," must be sought for in the rationalising of intelligence, that is, not merely its wider diffusion, but the clarification, the illumination, of the intellect, so that it may become conscious of the higher aims of human life and make choice of the best means of realising them. The result will be "the general differentiation of the faculties and refinement of the mental and moral organisation of the race," which the author speaks of as accompanying the development of self-consciousness, and it is merely a continuance of the process which Schopenhauer, in a passage quoted in the work under review, declares to originate the philosopher, the artist, and the poet, who are led to the contemplation of the world by the clearness with which they become conscious of it through the illuminating influence of *Besonnenheit*.

Much might be said with reference to Mr. Ward's opinion as to the action of

natural selection and the effect produced over organic existences by intellect in removing competition. We prefer to make a few remarks in relation to his view as to the origin of the intellect. He accepts, with reserve, Schopenhauer's statement that the intellect is an accident, and yet "it had a natural origin and was brought forth as a means of advancing nature's ends"; although it finally took upon itself to counteract the law of nature, by opposing the competitive system, and replacing it by the law of reason. Here is a confounding of reason with intellect, which in its choice of means and motives is strictly competitive, and thus it is the former attribute which is supposed to be accidental. The fortuitous origin of reason is consistent with Mr. Ward's earlier declaration that "while there is a cause for all things there is no intelligent reason why anything should be as it is." The initiative which led to "the plan of structure of organic forms" is said to be fortuitous. But how can a *plan* be regarded as in any sense fortuitous, and the very fact insisted on by the author, as being in favor of the meliorism which he rightly considers to form the true state rather than either optimism or pessimism, that all nature is a domain "of rigid law, of absolute impartiality," proves that his notion of fortuity is erroneous. Nature is organised throughout, and her structural arrangements are the necessary results of the operation of the principles which have governed the whole course of evolution from its very beginning.

The defects here pointed out do not, curiously enough, affect Mr. Ward's general position, which is that the amelioration of the social condition of humanity must be sought for in the exercise of man's highest mental attributes. Man has been able, through the enlightening influence of reason, to subjugate much of external nature to his purposes and thus to change the character of his environment, and now he has to do the same for his own nature, which requires effective control and guidance. The individual and society act and react on each other, and the author is right in making psychology the basis of sociology and in seeing its principles operative in the process of social evolution. While not prepared to endorse all Mr. Ward's statements, we think that his views in relation to that process and the purposes of sociology are, subject to the remarks, already made, incontrovertible, and in our estimation, therefore, his work deserves attentive study.

C. STANLAND WAKE.

EMPIRISCHE PSYCHOLOGIE NACH NATURWISSENSCHAFTLICHER METHODE. Von Moritz Wilhelm Drobisch. Hamburg and Leipsic: Leopold Voss. 1898.
Pages xvi, 355. Price, 6 M.

The *Empirische Psychologie* is not a new book, but the reprint of a work which appeared in 1842. The publisher informs the reader that the new edition has been published on the basis of an agreement with the heirs, and that the late Professor Drobisch who would not permit an unrevised edition to appear during his life-time had expressly given his consent to a posthumous reprint. The reader, accordingly, must bear in mind that the book is an historical document, and not a psychology,

which has been worked out in details and is thoroughly up to date. This granted, the book is of sufficient importance and interest to warrant its being made again accessible to students of psychology, after a lapse of forty years.

The main fault of the present edition (aside from the usual shortcomings of European books, i. e., absence of an Index) is the omission of an introduction by a competent man. It is true that psychologists are supposed to know who Drobisch is, and professors of psychology can tell us all about him, but there are also students and beginners to whom the book might be of use, and they cannot be expected to be fully informed on the history of German psychology. Reference books on the subject are still rare; for instance, Dr. Max Dessoir's *History of Modern Psychology in Germany* does not as yet give any information concerning Drobisch, because its first volume only has appeared, covering mainly the eighteenth century; and the Index of the *Encyclopædia Britannica* does not contain the name of Drobisch.

Drobisch, nevertheless, plays an important part in the development of psychology as a science; he is not the inaugurator and founder of the new method, but its strongest and most competent advocate. The founder of modern psychology is Herbart, for he pronounced for the first time, clearly and in an outspoken opposition to his predecessors, the fundamental truth that consciousness is an activity, *eine Thätigkeit*, which overthrows at once the proud system of the soul as an entity endowed with faculties. Herbart's psychology, it is true, is not an absolutely new beginning; he took up the thread of investigation where Kant had left it. Kant did not write a psychology, but his *Critique of Pure Reason* and his other works contain many valuable suggestions, which were worked out, in the spirit of his philosophy, by Reinhold in the latter's *Neue Theorie des Vorstellungsvermögens*, where the characteristic features of psychic life were defined as consciousness. On the basis of Reinhold's psychology Fichte built up his system of the ego, drifting slowly away from empiricism into the deep sea of metaphysicism, where he discovered the absolute ego, a natural product of (we may call it a compromise between) his one-sided idealism and universalism, recognising the reality of the not-me.

Schelling, dazzled by Fichte's conception of consciousness as an ego-unity, wrote a book on "the history of consciousness," full of ingenious speculations of that nature-philosophy which is not yet a philosophy of the natural sciences, as it is not yet free from fantastic elements. Carl Gustav Carus worked in the same line with Schelling; partaking of the latter's faults, but being more familiar with the physiological facts then known, he is more reliable in his studies, and his books on the *Symbolism of the Human Body*, his *Physic* and his *Psyche*, contain many suggestions that still deserve our attention. In Hegel the method of a one-sided anti-empirical psychology culminated. However ingenious his system is, however bold and comprehensive, it lacks the stimulus of empirical investigation. It is too dictatorial to be of any practical use, and spider-like it spins its conceptions of life

and world out of self-consciousness, which is the latest phase in the development of the absolute. Hegel's psychology is a retrospect of the evolution of man, as a conscious rational soul, rising from unconscious beginnings through contradictions to a synthesis of position and negation like the realisation of a living logical syllogism.

Against this dictatorial conception of the purely logical in the evolution of the human soul, Beneke, Fries, and Herbart raised a protest which, however, was not heeded by the enthusiastic Hegelian school, and had to fight its way until Hegelianism broke down through its own hollowness. Hegelian philosophy proved to be a plant of most luxuriant growth, full of great promise, but there were no fruits. It prospered for a season in Germany, i. e., about forty years, until 1860; then the axe was put to its roots, leaving ample time to Hegel's admirers to transplant sprigs of it in England and America, where they became acclimatised and are, within certain circles, still flourishing.

We need not here review Herbart's work and merits, for they are sufficiently known in this country. He is the founder of modern psychology, because he first of all attempted to reduce the facts of soul-life to exact scientific terms that could be expressed in mathematical laws. He started on the right path, in which Weber and Fechner followed, establishing by slow and painstaking work psychology as a natural science. There is perhaps a greater need of calling attention to the shortcomings of Herbart's psychology, to those points which by his successors had to be abandoned, but are still beclouding the minds of his disciples in educational lines.

Herbart was not only a psychologist; he was also an educator and a philosopher, and his pedagogy is born of the heart. He was an enthusiast who appreciated the importance of education. Herbart's psychology broke down the traditional superstition of the all-sufficiency of the ego with its faculties, and his application of mathematics led him to conceive the psychical organisation as a mechanism which made the situation clear and rendered it possible to formulate the problems of psychology in definite and solvable propositions. But with all this, Herbart still believes in an actual unity of consciousness, which in his psychology plays the part of the soul; but this unity has lost all the substantiality with which the old metaphysics endowed it, and has dwindled down to the shape of a mathematical point. It is difficult even now, as the case of Flügel and his controversies prove, to convince a Herbartian that the conception of a punctual soul is as metaphysical as a substance-soul. The term Herbartian is to-day applied to those disciples of Herbart who swear by the master's word and blindly accept, together with the valuable merits of his psychological and educational work, the antiquated views of his metaphysics.

Drobisch was not a psychologist by profession, but a mathematician and a philosopher; and this fitted him all the better for an appreciation of Herbart's merits as a psychologist, which he evinced in a book published in 1834 under the modest title of *Beiträge zur Orientirung über Herbart's System der Philosophie*, which

was followed in 1842 by his *Empirische Psychologie nach naturwissenschaftlicher Methode*, which is distinguished by directness and an utter absence of metaphysical methods, even to the exclusion of his master's theory of the point-soul, and his *Grundlehren der mathematischen Psychologie*, 1850.

Among other works of Drobisch we may mention his *Grundsätze der ebenen und körperlichen Trigonometrie*, 1825; *Philologie und Mathematik*, 1832; *Grundsätze der Lehre von den höheren numerischen Gleichungen*, 1834; *Neue Darstellung der Logik nach ihren einfachsten Verhältnissen*, 1836; *Grundlehren der Religionsphilosophie*, 1840.

Drobisch's psychology is remarkable for its simplicity, and it may still prove valuable for teachers in sketching out their course of lectures, filling out the various chapters with later experiments and other materials of a more recent date. For school purposes the book might be condensed, as many subjects admit of a briefer treatment; for universities it ought to be considerably enlarged; but we must repeat once more, in fairness to the author, that the book cannot serve as a text-book of to-day, but must be considered as an historical document in the evolution of psychology.

GESCHICHTE DER NEUEREN DEUTSCHEN PSYCHOLOGIE. Von Max Dessoir. Zweite völlig umgearbeitete Auflage. Erster Halbband. Berlin: Carl Duncker. 1897. Price, Mk. 8.

Max Dessoir of the University of Berlin is not only a practical worker in experimental psychology, but also an inquirer, a student of the past, a man of scholarly tendencies,—a combination which pre-eminently fits him for writing a *History of Modern Psychology in Germany*. He is as much at home in the subject as any one is and is personally acquainted with all the psychologists of to-day. He qualified himself in a number of historical investigations in kindred lines, all of which are distinguished by an ability for presenting the materials under discussion in a fascinating way. We have only to remind our readers of his essays on "The Psychology of Legerdemain," and "The Magic Mirror." The former appeared years ago in *The Open Court*, and the latter in *The Monist* (Vol. I., No. 1).

The present book is full of details and promises to become the standard work on the history of modern psychology in Germany. It begins with a short sketch of antique and mediæval psychology and points out the preparations for modern psychology in Paracelsus, Cardanus, Telesius, Campanella, Pomponatius, Vives, and others. A new epoch begins with Leibnitz whose name accordingly heads the first chapter after the introduction.

Dessoir points out three characteristic features in Leibnitz, an artistic disposition, a mathematical conception, and great receptivity. He makes the soul the basis of his world-conception, for, says he, any one who comprehends the soul has found the solution of the world-problem. Details concerning Leibnitz's views of

perception and apperception, his theory of monads, and the work of his disciple Tschirnhausen are treated in the second part of the first chapter.

The heir of Leibnitz is Thomasius, with a psychology strongly tinged with individualism. Thomasius is the same who is better known as the abolitionist of witch-prosecution, a man of practical insight and one in contact with real life. Wolff, however, who follows both in time, filling the third chapter, represents a systematisation of the traditional views of the schools.

Dessoir's treatment of the school of Wolff (chapter 4) and of his adversaries (chapter 5), as well as the eclectic psychologists (chapter 6), will be very welcome to the reader, since this is a period in the history of modern psychology which is least known and the sources of which are almost inaccessible in the libraries of America.

Empiricism in German psychology begins under the influence of English, and later on also of French thinkers, whose thoughts subsequently told so strongly on Kant and left their traces in all his followers. The first volume breaks off with the year 1777, the date of publication of Tetens's philosophical essays, which indicated the new spirit of the age to be realised by Kant, who at that time was preparing his essay of habilitation.

We hope to resume the discussion of Professor Dessoir's work on its completion, or on the appearance of its successive volumes. P. C.

KARL MARX AND THE CLOSE OF HIS SYSTEM. A Criticism. By *Eugen v. Böhm-Bawerk*, Austrian Minister of Finance, and Honorary Professor of Political Economy in the University of Vienna. Translated by Alice M. Macdonald. With a Preface by James Bonar, M. A., LL. D. New York: The Macmillan Co. 1898. Pages, 221. 8vo. Price, \$1.60.

The economic ideas of Karl Marx have retained so great an influence among the accepters of socialism, notwithstanding the repeated attempts to prove their falsity, that it is difficult to believe them to be erroneous and the system based on them without logical justification. And yet the author of the present work would seem to have demonstrated such to be the case. Herr von Böhm-Bawerk is well known to American and English students of economic science as the author of *Geschichte und Kritik der Kapitalzinstheorien*, and he is well qualified to criticise the third volume of Marx's *Capital*, which appeared eleven years after the author's death and more than thirty years after the publication of the first volume. Marx's third volume was looked forward to with great interest both by his adherents and his opponents, because it was expected to give his solution of a question of fundamental importance. This question had been raised by Marx himself in his first volume, but instead of answering it he promised to do so in a succeeding part of his work. It was contained in the admission that the law, that surplus value is in proportion only to the variable part of capital—the part paid in wages—"clearly contradicts all *prima facie* experience." This contradiction was declared to be

only seeming, but when the second volume of *Capital* appeared without the promised solution, the suspense became trying, although its editor, Friedrich Engels, the author being already dead, asserted positively that it was contained in Marx's manuscript. He went further and challenged the followers of Rodbertus in particular, in the interval before the appearance of the third volume, to solve from their own resources the problem "how, not only without contradicting the law of value but even by virtue of it, an equal rate of profit can and must be created." The effect of this challenge furnishes striking testimony to Marx as a thinker, as mentioned by Herr von Böhm-Bawerk, who states in his Introduction that economists of various schools attempted to penetrate the mystery in which Marx's views were shrouded. There was even a regular prize essay competition on the "average rate of profit" and its relation to the "law of value." No one succeeded in carrying off the prize, although, as pointed out by Dr. Bonar in the Preface to the present volume, Professor Lexis gave substantially the same answer as that supplied by Marx's third volume. This was published in 1894, and the problem and its solution are considered by von Böhm-Bawerk in that part of his *Criticism* which treats of "The Question of the Contradiction," which he shows to be evaded instead of the contradiction itself being got rid of. Important as is this portion of the author's work, still more so is that which has for its topic the "Error in the Marxian System," and as this is of more general interest than the former we will give consideration to it first.

As is well known, the fundamental thesis of Marx's system is that labor is the real source of value. This view was taken by the earlier economists Smith and Ricardo, though without furnishing any proof of its truth, although they found evidence of it satisfactory to their own minds in an assumed natural state, "an idyllic state of things where labor and value were one." Such an assumption agreed so well with the socialistic tendencies of Marx that he accepted it unconditionally, and it became to him a matter of earnest conviction. For his system, however, he had to supply formal proof of the truth of the statement that value is derived from labor, and this he did "in the form of an abortive dialectic, more arbitrary and untrue to facts than has probably ever before been known in the history of our science." It is impossible to follow Herr von Böhm-Bawerk through all the stages of his exposure of Marx's errors, but that which deals with the fundamental proposition of the system may be noticed in some detail. This proposition is, that the exchange value of commodities finds its origin and its measure in the quantity of labor incorporated in the commodities. Marx offers no proof of its truth drawn from experience, and he could not have done so, for "the reasoning of the third volume proves that he was quite aware of the nature of the empirical facts, and that they were opposed to his proposition. He knew that the prices of commodities were not in proportion to the amount of incorporated labor, but to the total cost of production, which comprises other elements besides." Nor does Marx adopt the psychological method and endeavor to establish the truth of his proposi-

tion by reference to the motives which, on the one hand, govern people in the determination of exchange prices, and, on the other hand, guide them in their co-operation in production. He prefers the simpler plan of taking Aristotle's idea that "exchange cannot exist without equality, and equality cannot exist without commensurability," and expanding it by conceiving the exchange of two objects under the form of an equation, and inferring "that 'a common factor of the same amount' must exist in the things exchanged and thereby equated." He then proceeds to search for this common factor, and by the exclusion of all the properties possessed by the objects exchanged which cannot stand the test, he finds it to be labor and nothing but labor. This proof by "negative instances" fails, because Marx, in searching for the common factor, neglects the exchangeable goods which are not products of labor but are gifts of nature, such as coal-beds, stone-quarries, the soil, gold mines, etc. This narrowing of the sphere of exchangeable goods as a whole is ingeniously effected by the employment of the term "commodities," and he then proceeds to get rid of the competitors of labor as creators of value by affirming that a *value in use*, or a *good*, "has only a value because abstract human labor is stored up or materialised in it." The author has no difficulty in proving that the reasoning in support of this proposition is fallacious. And he shows that Marx was repeatedly forced to admit that there can be no exchange value where there is no value in use. He points out, moreover, that if the subjects of two paragraphs he quotes were transposed, as would have been the case if Marx had chanced to reverse the order of the examination which led to the exclusion of the value in use, labor would have been excluded in its stead, without the seeming-justness of the reasoning being affected. Marx's next proposition is that the value of different commodities is in proportion to the working time necessary to their production, and when arguing in support of it he asserts that "skilled labor counts only as concentrated or rather multiplied unskilled labor," which he justifies by reference to experience. The author has no difficulty, however, in showing that Marx's reasoning is in a circle, although its defects are so cleverly concealed as not to be noticeable by the ordinary reader.

One of the best portions of von Böhmer-Bawerk's book is that which immediately follows, in which he examines into the position accorded by Marx in his system to "competition," and his views in relation to supply and demand, which are said to cease to act when they balance each other. The falsity of this notion is exposed and Marx's statements made to exhibit so many contradictions that his critic is led to the conclusion that the system is not in touch with the facts.

But it is time to say a few words in relation to "The Question of the Contradiction" already referred to. This question the author deals with in an elaborate manner, examining in detail all the arguments direct and indirect in favor of the position that, in the last resort, the Marxian law of value determines the prices of production, showing that they are utterly inconclusive and that the contradiction remains as strong as ever. That Marx's theory of value is not consistent with ac-

tual experience is admitted by Werner Sombart, to a consideration of whose apology for the Master the last chapter of the present volume is devoted. We must leave this to the reader, who will find in it much to support the criticism of Marx's system contained in the earlier chapters. The value of the work in its English form is increased by the condensation of the argument given in Dr. Donar's Preface and by its reference to noteworthy passages. With the quotation of one of these this notice may be concluded. After remarking that socialism, neither practical nor theoretic, will certainly not be overthrown with the Marxian system, the author continues: "As there was a socialism before Marx, so there will be one after him. "That there is vital force in socialism is shown, in spite of all exaggerations, not "only by the renewed vitality which economic theory has undeniably gained by "the appearance of the theoretic socialists, but also by the celebrated 'drop of "social oil' with which the measures of practical statesmanship are nowadays "everywhere lubricated, and in many cases not to their disadvantage." In conclusion we may say that the translators of the work deserve credit for its appearance in English dress, and it will doubtless many find readers among English-speaking peoples.

C. S. WAKE.

SOME PHILOSOPHY OF THE HERMETICS. Los Angeles, Cal.: B. R. Baumgardt & Co. 1898. Pages, ii+109. Price, \$1.25.

SOME MORE PHILOSOPHY OF THE HERMETICS. Los Angeles, Cal.: B. R. Baumgardt & Co.; New York: Alliance Pub. Co. 1898. Pages, viii+232. Price, \$1.50.

These two books contain rhapsodies on Hermetics, Philosophy, Faith, Imagination, the Devil, etc. The first book begins as follows:

"Nature has a way of concealing and revealing. She tells half her story out "in the sunshine in a loud voice, and the other half in whispers underground.

"She is coy like a coquette, and stern like a judge. She excites curiosity in "the student, and dread in the debauchee.

"She holds the man of science to her breast, but is dumb to the lover of "pleasure. She scorns the victim of priestcraft and repudiates the supernatural."

The last chapter which is on magic closes as follows:

"Would you be a magician, stir up the smoldering coals at your own fireside. "Begin to burn. Feel your blood hot in your veins. Warm yourself with memories of sun-tinted dreams. Pray—pray—pray at the shrine of the Sphinx."

The Hermeticism of the Middle Ages is repudiated. The author says:

"The absurdity of the Hermetic of the Middle Centuries would be laughable "were it not so pathetic. When he speaks of sulphur and mercury and so forth "and so on, his pages in print appear more like the ravings of a lunatic than any "thing else. To pass as a harmless crank was his only hope of living at all, once "upon a time. 'But to-day,' you say, 'there is no danger, why keep up this absurd symbolism?' We reply, partly from the association of ideas, which, in a

"way, has become pleasant to us, for the past is at our backs and its memories are sacred, and partly from the first reason given, which is, that man is by constitution hermetic and tells only so much of his story as the world is willing to receive. A certain symbolism, in guise of parable and illustration, was used by the great Masters of philosophy and religion—Jesus and Gautama, to say nothing of the Masters in Egypt—ere written history began. The symbol condenses, and carries a deal of meaning along with it that pages can not express."

From the prospectus of the book we learn that Professor Jordan, President of Stanford University (who presumably is a personal friend of the author) speaks of the book as follows:

"It is full of sound wisdom thrown into a striking literary form which seems to hide the commonness of its 'common sense.' It is a book to be encouraged, and it ought to be a practical help to many in the conduct of life."

President Jordan is commonly regarded as a clear thinker, and his praise of the Hermetic philosophy may be a surprise to many; for these books can be appreciated by mystics only.

BOUDDHISME: ÉTUDES ET MATÉRIAUX. Adikarmapradīpa Bodhicaryāvatāraṣṭka.

Par Louis de la Vallée Poussin. London: Luzac & Co. 1898. Pages,

417.

This book in large quarto of some four hundred pages contains the text of the Adikarmapradīpa Bodhicaryāvatāraṣṭka, a book of Tantrik Buddhism, which teaches the vanity of both world and thought, the uselessness of ritual, and seeks salvation in the triple abhyāsa, a kind of yoga, by which the guru (or disciple) realises his identity with the Buddha. Professor Poussin follows MS. 69 of the Royal Asiatic Society, and accompanies his edition with brief variants, suggestions, and references. The most interesting part of the book for the reader who is not a specialist, will be the introduction, pp. 1-161, in which the author presents a new view of Buddhism in opposition to the traditional view represented by Oldenberg and his colleagues, who as a matter of principle limit their investigations to the three baskets of the southern schools, commonly called Hināyana, or small vessel of salvation. Professor Poussin claims, and he defends his position with good arguments, that the unity of the southern Buddhism is a fiction, and that the broader school of northern Buddhism, commonly called the Mahāyana or large vessel of salvation, has the same title to an historical consideration as the narrow doctrines of the southern school. Gautama was not the founder of asceticism, but on the contrary an opponent of its narrow system of salvation. He did not limit his instruction to the order of monks, but accepted willingly and gladly great numbers of lay disciples. We cannot doubt that there were great varieties of Buddhist congregations all of whom acknowledged the authority of the Buddha as their guide and master in some form or another, and the Hināyana was one of them, but by no means the only one, and probably not a very large fraction of the whole Buddhist community in India.

But this Hināyana, which consisted exclusively of monks, was severer in discipline and more rigid in doctrine than the other schools and assumed at the same time an authority which mislead historians and Pāli scholars to regard it as the sole, original, and unadulterated Buddhism. Their conception of the situation, however, leaves gaps in the history of Buddhism which throws our comprehension of the development of its churches, especially in Nepal and Burmah, but in other countries also, into confusion.

Professor Poussin's work is not yet complete. He gathers materials of which the present edition of ancient MSS. is a beginning only. We hope to see more from his pen and do not doubt that his labors will bring valuable materials to the notice of scholars and become a safe foundation for a critical and truly scientific history of Buddhism, which as yet does not exist. P. C.

LA MATHÉMATIQUE. PHILOSOPHIE—ENSEIGNEMENT. By C. A. Laisant. Paris: Georges Carré and C. Naud. 1898. Pages, 286.

We have in this work a new contribution to the philosophy of mathematics,—a field which has been much worked, but in which considerable labor yet remains to be done. M. Laisant has given us a clear and simple book, which makes no pretension to erudition or to metaphysical cloudiness; he has addressed his utterances not to finished students of the higher branches of mathematics nor to persons who have no knowledge whatever of this science, but to those who have already studied the subject, who are teaching and applying it, but have not had the opportunity of making it a specialty. Mathematics has grown to such proportions in the nineteenth century, and research in each branch has been pushed so far, that it is impossible for any single individual to command it in its full extent; much less, then, is it to be supposed that instructors and students who have not made this science their life-study can be in the possession of secure and comfortable ideas regarding its actual status and its possibilities. It is to meet the wants of this class that the book has been written.

The relations between philosophy and mathematics are first discussed, and the opinion of Leibnitz quoted that "one cannot go to the bottom of philosophy without mathematics, that one cannot go to the bottom of mathematics without philosophy, and that without both one cannot go to the bottom of anything." The work is divided into three parts, viz., (1) the philosophy of pure mathematics, (2) the philosophy of applied mathematics, and (3) instruction in mathematics. The first part deals with the classification of the mathematical sciences, arithmetic, arithmology, algebra, the infinitesimal calculus, theory of functions, geometry and analytical geometry, and pure mathematics; the remaining parts are correspondingly divided. The author believes in the experiential origin of mathematics, contending that the only difference between this science and the other sciences is that it borrows a minimum of notions from experience, the rest being a purely logical development. There are interesting remarks upon the object and utility of the mathemat-

ical sciences; the discussion regarding the classification of the various subdivisions of mathematics involves many instructive points, and constitutes a broad, general survey of the present state of mathematical research.

The chapter on instruction is also interesting and suggestive. As to the amount of primary instruction which should fall to the share of pupils, the author believes that the first elements of the sciences of number and of space should be inculcated, but not in a mechanical manner; the course should be an elastic one, the general outlines of which only should be determined. The theoretical instruction, further, should be supplemented by applications to real facts,—which applications should be few and happily chosen, rather than many and indiscriminate. M. Laisant contends that if the elements of arithmetic, algebra, and geometry were disengaged from the mass of parasitic propositions which envelop them, if they were reduced in quantity so as to embrace only the leading ideas and the essential methods, enough time would be gained and sufficiently clear ideas would be imparted to enable instructors to include in all courses leading up to University examinations the elements of analytical geometry and the infinitesimal calculus. This amount of instruction, which is thoroughly reconcilable with the requirements of a course of general culture, and which does not transcend the ordinary intelligence, is what should be expected of every educated person.

While the author makes no pretension to profundity, and does not expect that his work will throw great light on the metaphysical foundations of mathematics, his ideas are impregnated with common sense and are clearly stated; instructors and students will both profit greatly by a perusal of the book. There is a bibliography and an index.

We cannot, in conclusion, refrain from complimenting the publishers on the elegant taste they have displayed in the letter-press and especially in the binding of the volume.

DAS PRINCIP DER ERHALTUNG DER ENERGIE UND SEINE ANWENDUNG IN DER NATUR-LEHRE. Ein Hilfsbuch für den höheren Unterricht. By *Hans Januschke*.
Leipsic: B. G. Teubner. 1897. Pages, x, 456. Price, bound, 12 M.

The present work is another evidence of that reform in the methods of instruction which has replaced the old dogmatic method by a mode of exposition predominantly psychological and historical in character; it has also been noticeably influenced by the philosophical movement in science which found its expression in the works of Helmholtz, Kirchhoff, Dühring, Mach, Hertz, Boltzmann, Ostwald, and others. The aim of the author has been to demonstrate, by practical exposition, the validity of the principle of the conservation of energy in every branch of physics, to lay in this way a unitary foundation for the whole subject, and so to satisfy the educational requirements of economy and consistency. He has treated his material according to the form which it has historically taken, beginning with mechanics and hydrostatics, and proceeding through heat to electricity, magnetism

and light. There is a brief introduction to the work on the subject-matter and methods of physics, and also a historical sketch of the origin and development of the principle of the conservation of energy,—sections which convey much interesting information, but contain nothing very original. The work is intended as a companion book to advanced courses in physics, and the development of the several subjects is largely couched in the language of the Calculus,—which is, however, of a very simple character. An excellent feature, which is entirely new in German books of this character, is the appending to each section of a number of examples for practice. Also, the existence of an index and the fact that the book is bound, are pleasing indications of a revolution in German book-making. The letter press in general is good, and upon the whole the author and publisher have produced a work of value.

L'ANNÉE PSYCHOLOGIQUE. Publiée par *Alfred Binet*. Avec la collaboration de *H. Beaunis, Th. Ribot, MM. Bourdon, Courtier, Farrand, Flournoy, Philippe, Vaschide, and Warren*. Quatrième année. Paris: Schleicher Frères, 1898. Pages, 839. Price, 15 francs.

The *Année psychologique* is now in its fourth year, and the indefatigable industry of its editor and of its collaborators has made it one of the most useful of the year-books of science. It is divided into three parts, the first of which consists of original memoirs; the second of critical reviews, digests and synopses of the most important works and articles published on psychology and kindred subjects in the year 1897; third, of a bibliographical list and description of *all* the works published on the subject during the same year. This last list has been compiled by two Americans, Drs. Warren and Farrand, and is the same as that of the *Psychological Index*. We find in this list that the number of works published on psychology and kindred subjects for the year 1897 was 2465. Of course, this includes such subjects as brain-physiology and much that properly belongs to medicine; for example, looking over the books which have been reviewed, we find the following subjects: psychogeny, comparative and individual psychology, anatomy and physiology of the nervous system, sensation, consciousness, attention and intellect, emotions, movement and volition, abnormal and pathological psychology. The original memoirs are mostly contributions of the editor, who is director of the Laboratory of Physiological Psychology at the Sorbonne, in collaboration with M. Vaschide; the two remaining contributors of original memoirs are MM. Bourdon and Leclère. The memoirs of MM. Binet and Vaschide deal principally with the psychology of school children, and record the results of experiments testing muscular force, respiration, circulation, etc., etc.